MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY

Vol. 98 No. 3

MEETING OF 1938 JANUARY 14

Dr. H. Spencer Jones, President, in the Chair.

The election by the Council of the following Fellows was duly confirmed :—

Robert d'Escourt Atkinson, M.A., D.Phil., Royal Observatory, Greenwich, London, S.E. 10 ;

Edwin A. Baker, D.Sc., Royal Observatory, Edinburgh ;

William Henry Barton, jun., Associate-Curator of Astronomy and of the Hayden Planetarium, American Museum of Natural History, New York City, U.S.A. ;

John Ralph Bazin, 84 Hendon Lane, London, N. 3 ;

Miss Cicely Mary Botley, F.R.Met.Soc., 17 Holmesdale Gardens, Hastings ;

Hermann A. Brück, Solar Physics Observatory, Cambridge ;

George Arthur Bull, B.Sc., F.R.Met.Soc., Meteorological Office, Royal Air Force, Felixstowe, Suffolk ;

Rev. Reginald Henry Consterdine, Lisadel, Wilmslow, Manchester ;

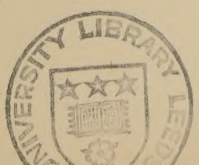
E. T. Holdaway, Government Astronomer for Time Keeping, Queensland Observatory, Brisbane, Australia ;

Basil Munroe Holden, B.A., 15 Hampstead Lane, Highgate, London, N. 6 ;

Clifford George James, Fairey Aviation Co., Ltd. (Seaplane Section), Hamble, Hants, and 5 Longmore Avenue, Woolston, Southampton ;

Eric Mervyn Lindsay, M.A., M.Sc., Ph.D., Director of the Observatory, Armagh, Ireland ;

Miss Flora M. McBain, H.M. Nautical Almanac Office, Royal Naval College, Greenwich, London, S.E. 10 ;



Alexander Menzies, Royal Observatory, Cape of Good Hope, South Africa ;

Rev. W. O'Leary, S.J., Director of the Riverview Observatory, Sydney, N.S.W., Australia ;

Gordon Edmond Patston, 55 Woodbourne Avenue, Streatham, London, S.W. 16 ;

John Guy Porter, B.Sc., F.C.S., 13 Cleave Road, Gillingham, Kent ;

George Ormerod Rawstron, 216 Pullman Court, Streatham Hill, London, S.W. ;

Reginald Campbell Smith, 99 Broomhill Drive, Glasgow, W. 1 ; and

Joseph Bernard Godsell Turner, Royal Observatory, Cape of Good Hope, South Africa.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

Miss Madge Gertrude Adam, University Observatory, Oxford (proposed by H. H. Plaskett) ;

Rev. George E. Harrison, The Manse, Smeaton, Victoria, Australia (proposed by John Brittain) ;

Rev. Joseph Lynch, S.J., Dept. of Physics, Fordham University, New York City, N.Y., U.S.A. (proposed by H. H. Plaskett) ; and

Lieut.-Col. J. E. Nicholson, The Gables, Somerset Road, Douglas, Isle of Man (proposed by D. R. Fotheringham).

Ninety presents were announced as having been received since the last meeting, including:—

Francis R. Johnson, "Astronomical Thought in Renaissance England: a Study of the English Scientific Writings from 1500 to 1645" (presented by the Oxford University Press).

The President announced that the Gold Medal of the Society had been awarded by the Council to Dr. William Hammond Wright, Director of the Lick Observatory, for his studies of the spectra of gaseous nebulæ and of novæ and for his work on the photography of planets in light of different colours.

The President further announced that Jackson-Gwilt Medals and Gifts had been awarded by the Council to Mr. P. M. Ryves for his observations of variable stars and other astronomical work, and to Mr. F. J. Hargreaves for his contributions to astronomy.

The President made the following announcement :—

The Society has received a gift of 2500 dollars (which realized £500, 15*s*.) from Miss Maude S. Capps, of Bloomington, Illinois, under the following circumstances. Mr. A. G. Stillhamer, F.R.A.S., died on 1936 February 5, leaving all his property outright to Miss Capps. But on account of certain

personal memoranda and conversations between Mr. Stillhamer and Miss Capps, the latter resolved to carry out what she believed to have been Mr. Stillhamer's wishes by the payment to the Society of the sum of 2500 dollars in cash. The conditions of acceptance are that the principal is to remain intact in the endowment fund of the Society, and the income therefrom to be used annually to pay expenses of scientific research or to purchase definite scientific instruments for use in scientific instruction or research.

An agreement on the above lines between Miss Capps and the Society has been signed and sealed, and the money has been invested in $3\frac{1}{2}$ per cent. War Stock.

The President further announced that the Council had decided to allocate a portion of the Spencer Bequest to research, and that a committee had been appointed to consider applications for grants from the Spencer Bequest and from the Capps Gift (Stillhamer Bequest). Any Fellow intending to apply for a grant from either of these sources was requested to do so as early as possible.

Miss Elizabeth Williamson spoke on her recent translation of Ptolemy's *Almagest*.

CELESTIAL PHOTOGRAPHS

The following have been added to the list of Celestial Photographs reproduced by the Society for sale to Fellows :—

R.A.S. Serial No.	Ref. No.	
502	F 57.	Comet Finsler. 1937 Aug. 7–8. Blumbach, Norman Lockyer Observatory. 5·5-inch Zeiss Triplet, *f.l.* 29 inches. Exposure 5 h.
503	F 58.	Comet Finsler. 1937 Aug. 8–9. Blumbach, Norman Lockyer Observatory. 5·5-inch Zeiss Triplet, *f.l.* 29 inches. Exposure 5 h. 21 m.

A COMPARISON IMAGE MICROMETER.

C. R. Davidson and L. S. T. Symms.

(Communicated by the Astronomer Royal)

PART I.—*Design*, by C. R. DAVIDSON

The micrometer is designed on the principle that the difference between two nearly equal objects is readily detected whilst the measurement of their actual dimensions is more difficult.

In the observation of double stars the filar micrometer is almost invariably employed. It is true that in a few special cases the interferometer may be used, but in the great mass of double-star measures the filar micrometer has been unchallenged. It has, however, some objections, two of which may be mentioned. The spurious disk of the star image is frequently, with faint stars, comparable in size to the width of the spider-line with which the bisection is made, consequently a bisection amounts to extinction, and this in the case of close and difficult pairs may introduce systematic error. Secondly, owing to variations of atmospheric refraction, the star image in the telescope is rarely steady; it is constantly dancing about its mean position, the amplitude of its gyrations amounting to 1 to 2 seconds of arc. The observer therefore has to catch the star as he can, and this is not at all easy.

Six years ago Mr. Hargreaves suggested a method by which the measurement of double stars might be improved.* It was in fact to produce in the field of the telescope an artificial double star which could be so varied in distance, position-angle and brightness as to match the real double star in these three respects. When this match has been realised the readings on the artificial mechanism would give a measure of the position-angle and distance of the double star under observation. He made a micrometer on this principle, described in the paper referred to, and measured a number of double stars with it. The measures so produced were greatly superior to those made under similar conditions with the filar micrometer, their internal accordance being remarkable.

That the micrometer might be tried out on a larger telescope it was sent to Greenwich to be applied to the 28-inch refractor. In its form as originally designed there were difficulties in fitting it to the telescope; also there were one or two points which it seemed desirable to modify, so that whilst the principle has been preserved the micrometer as now applied to the 28-inch is somewhat different in construction.

To produce the double star we have used a Wollaston prism. The idea is not new, as it was proposed by the Rev. W. Pearson more than a hundred years ago, though in a different manner.†

* *M.N.*, **92**, 72, 1931.

† *Memoirs of the Astronomical Society of London*, 1820 March.

A Wollaston prism is a rectangular parallelopiped made of two right-angled prisms of Iceland spar of the same refracting angle, but one is cut so that the refracting edge is perpendicular to the optical axis and the other so that the refracting edge is parallel to it. They are cemented together so as to make a parallelopiped, the optical axis of each prism being perpendicular to the direction of the incident light.

Thus in a normal cross-section of the united prisms the section *ABD* of one contains the axis, its direction being parallel to the face *AB*, whilst the section *BCD* of the other is perpendicular to the axis.

A ray incident normally on the face *AB* travels along the normal in the crystal as an ordinary ray with velocity v_0 and also as an extraordinary ray with velocity v_e. On reaching the interface the ordinary ray is refracted extraordinarily, for the planes of the two prisms are at right angles, whilst the extraordinary ray is refracted ordinarily. The two emerging rays are therefore equally deviated on opposite sides of the normal. Thus the ray from *S* on passing through the prism is divided into two rays, one in the

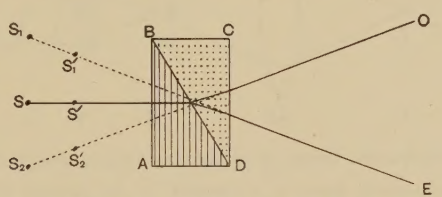


FIG. 1.—*The Wollaston Prism.*

(*The angles are exaggerated.*)

direction *O*, the other in the direction *E*, and the point *S* viewed through the prism will be seen as two points, S_1S_2, of equal brightness.

Now it will be noticed that if the prism is moved towards *S* (or *S* towards the prism) the separation S_1S_2 will diminish in exact proportion to the distance of *S* from the prism, becoming zero when *S* falls on the interface. As *S* passes to the other side of the interface the images cross and again separate.

Also if the Wollaston is rotated the double images rotate about their common centre.

If a Nicol prism be placed in the path of the rays with its polarising plane at right angles to the plane of *ABCD* only one of the polarised rays will be permitted to pass and one image only will be seen. If the Nicol is turned through 90 degrees the other ray will pass and the second image will be seen. In intermediate positions both images are visible, their relative brightness depending on the position of the Nicol. If the first Nicol is crossed by a second the total brightness of the images may be controlled by rotating the Nicols relatively to each other.

Here then is the principle on which an artificial double star has been produced—a double star which may be varied in distance, position-angle and magnitude, both total and relative.

To produce the point of light *S* an image of the filament of a 3-watt

electric lamp is projected on a small pinhole. An image of the illuminated pinhole is formed by a second lens, and this is the point from which the artificial double star emerges.

The arrangement is shown in fig. 2, which is self-explanatory.

That part of the optical system producing the primary image S is held in a tube which by means of a rack-and-pinion may be moved backward or forward longitudinally. Its position, read on a millimetre scale showing the position of S relatively to the Wollaston, furnishes the measure of distance.

The Wollaston and crossed Nicols are carried on a rotating circle, giving the position-angle.

The micrometer is bracketed to the eye end of the telescope so that it may be easily manipulated by the observer at the eyepiece. The beam

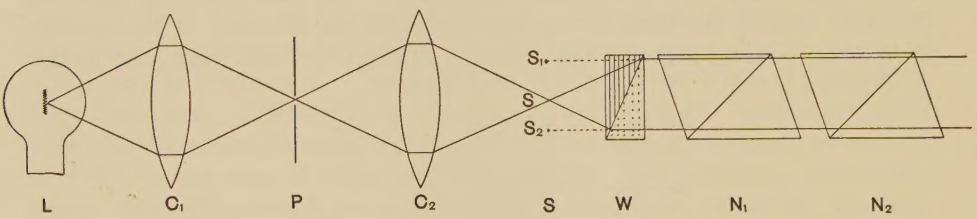


FIG. 2.—*Schematic Diagram of the Comparison Image Micrometer.*

An image of the electric lamp L is formed on the pinhole P. An image of the pinhole is formed at S. The beam then passes through the Wollaston prism W and the crossed Nicols N_1N_2. The image S is seen as two, S_1S_2, whose separation will vary as the distance from S to W varies.

from the micrometer is directed up the outside of the telescope tube on to a plane reflector 14 feet away. Sent back it falls on a diagonal prism which directs it into the telescope tube near the eyepiece. It now falls on a small object-glass of $11\frac{1}{4}$-inch focus and *via* a quarter-silvered cube the image of the artificial star is formed in the field of the telescope. The images so formed are minute points of light comparable with the nuclear images of the 28-inch at its best.

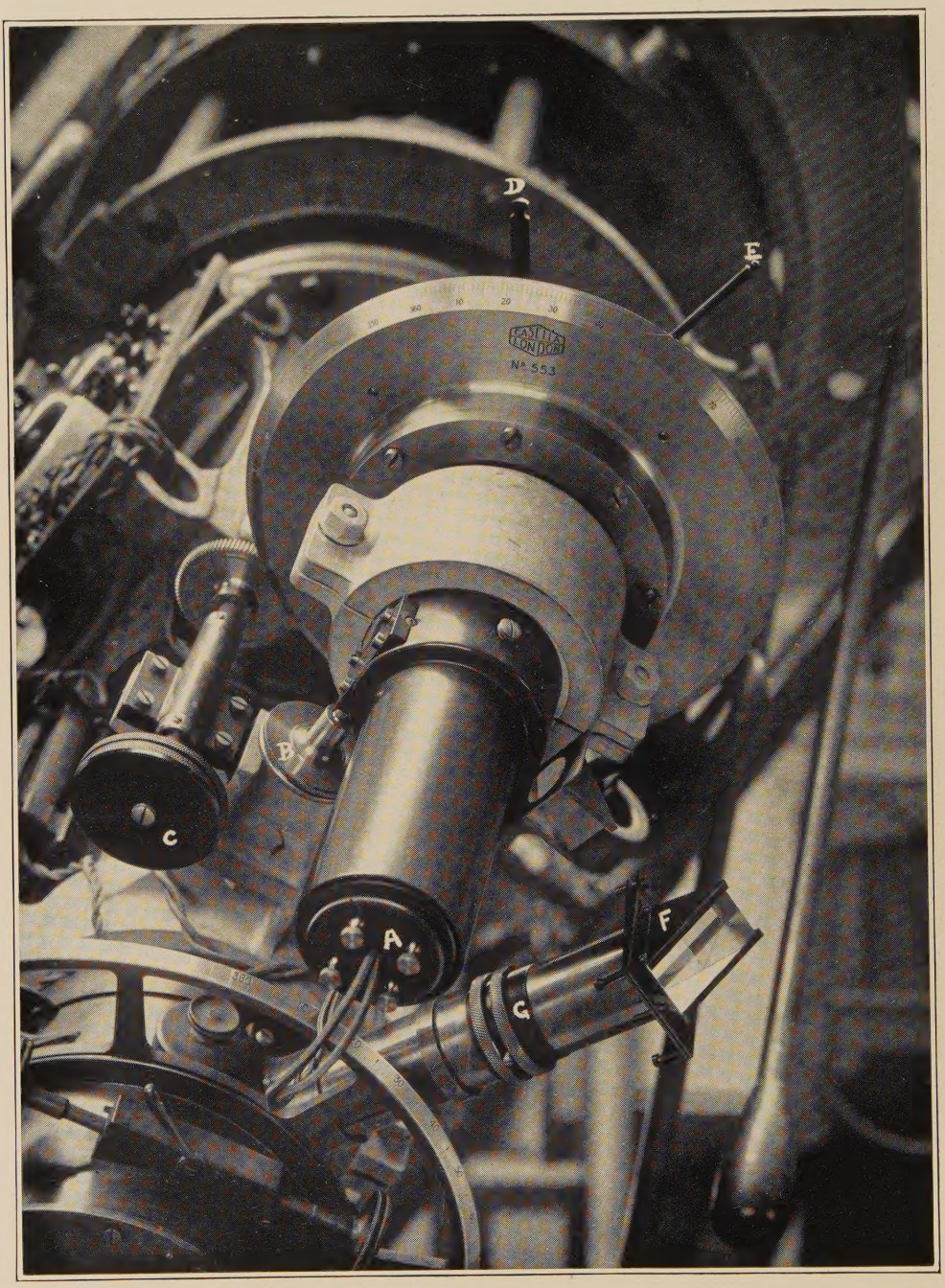
The deviation of the beams O and E (fig. 1) is a constant depending on the refracting angles of the component prisms. In the prism employed the divergence of the two beams is about 6 degrees, consequently a longitudinal movement of the Wollaston of 10 mm. causes a separation of the images S_1S_2 of about 1 mm. Viewed at a distance of 28 feet with an object-glass of $11\frac{1}{4}$-inch focus the separation becomes ·033 mm., which in the field of the 28-inch = ″·8 approx.,

or, 1 mm. on the scale = ″·08.

The actual value determined with the filar micrometer is

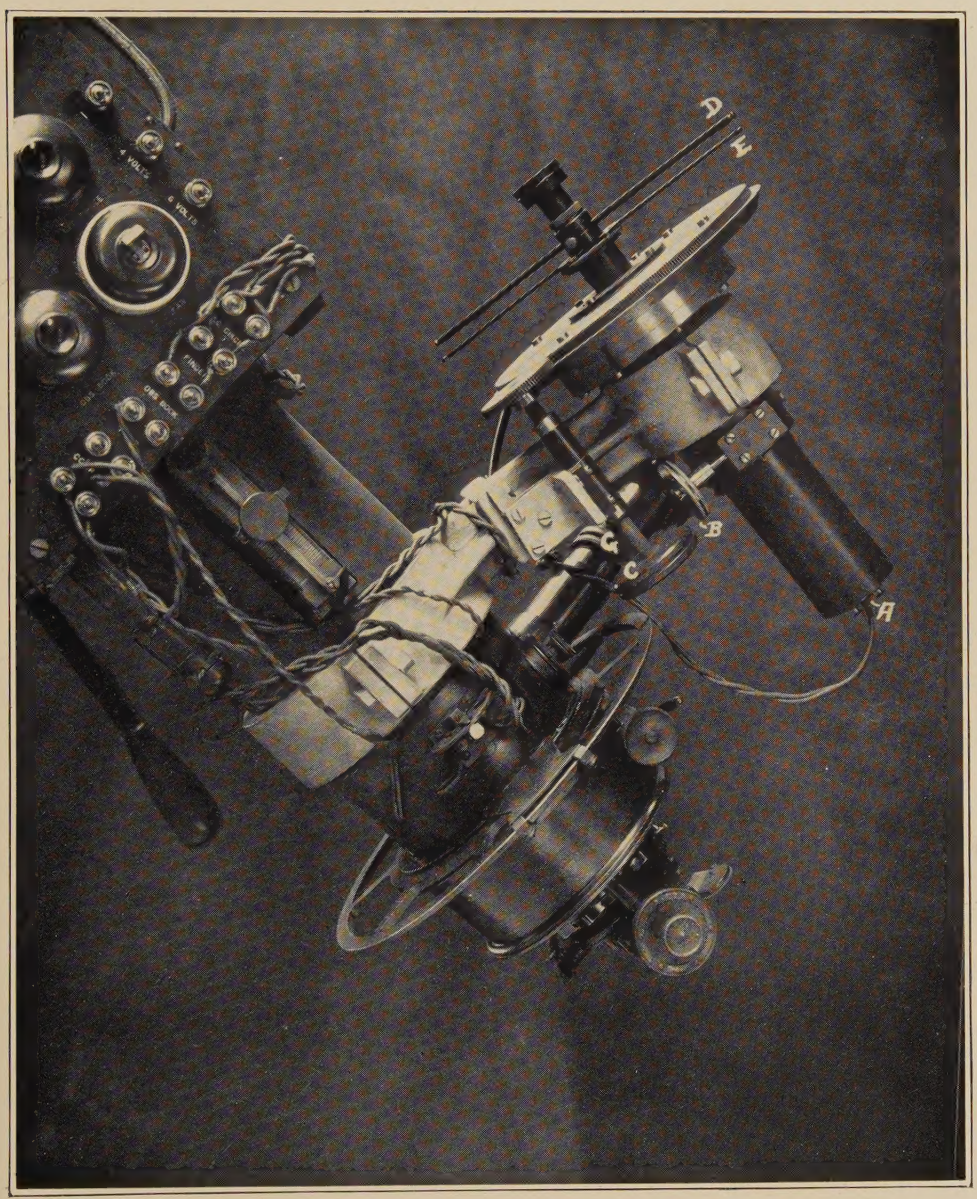
1 mm. = ″·0804.

It should not be overlooked that the image points produced by the Wollaston are really short spectra, whose angular dispersion between λ 4861



Comparison Image Micrometer mounted on 28-inch Telescope.

C. R. Davidson and L. S. T. Symms, A Comparison Image Micrometer.



A. Lamp.
B. Pinion to rack controlling separation of images.
C. Pinion rotating Wollaston prism and position-circle.
D, E. Controls of crossed Nicols.
F. Diagonal directing beam into 28-inch telescope.
G. Focussing rings of O.G. forming images in focal plane of 28-inch.

C. R. Davidson and L. S. T. Symms, A Comparison Image Micrometer.

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and λ 6563 is about 4 per cent. of the angular deviation, or of the separation of the images. The eye, however, is sensitive chiefly to those wave-lengths in the neighbourhood of λ 5500, and so long as the effective wave-length of the artificial star is not altered the scale remains unchanged. Actually with such separations as we are concerned with, the image is a point without trace of colour. The light source is a tungsten filament burning at a temperature of 2800° K. The colour temperature of a star image in the 28-inch may be taken as 5500° K. It was necessary therefore to introduce a blue filter to make the colours similar. In the result the artificial star, except that it is always steady and well-defined, is exactly similar to the real star.

Part II.—*Results of Observations*, by L. S. T. Symms

Method of Observation.—The real star image is brought near to the artificial star and a rough adjustment made for separation and position-angle. Then by manipulation of the crossed Nicols an exact match is made of the total and relative brightness of the components of the double. The images are next placed so that the four points are in a line and the final adjustment made for position-angle. The match for separation of the components is made with the images side by side, *i.e.* with the four points forming a rectangle or parallelogram. If the components are of unequal brightness it is found that the observation for position-angle cannot be made with certainty with the images side by side.

The comparison image is then rotated through 180° and the two components interchanged by moving the separation adjustment until they pass through each other. The matching of position-angle and separation is then repeated in the new position. This gives two measures of angle differing by 180° and one measure of separation corresponding to the double-distance method with the filar micrometer.

For a complete observation of a double star this process is repeated a second time and sometimes, if the measure appears difficult, a third time.

Results.—The micrometer has been in use since September 23 and we have gained enough material and experience of its performance to give the following report.

Wide and Close Pairs.—Dealing first with both wide and close pairs, the inconvenience of the continual movement of the image as a whole due to wind or variable refraction is greatly reduced, as was expected. The observation can be carried out without the exasperating waits for the image to cease from bouncing long enough to enable the wires to be placed on to it.

Much fainter stars can be measured as there is no need for either bright-field or bright-wire illumination. More is gained on this score than is lost by the inevitable loss of light by reflection and absorption at the quarter-silvered cube. The fact that the image is not covered by the wires in making the observation also allows of fainter stars being observed.

The diffraction disks of the artificial and real star images are so alike

that on a night of good seeing, but for the continual oscillation of the real star's images, they appear identical. In fact the images are so similar that it is impossible for the estimated separation to be seriously in error without the discrepancy being obvious. The great similarity of the images leaves no room for any personality in the observation of separation.

Wide Pairs.—On wide pairs (> 1 second separation), apart from the advantages just mentioned, there is not a great deal to choose between the new method of observing and the filar micrometer method. While the results obtained with the comparison image micrometer in the measurement of separation are much more self-consistent the probable errors in the position-angle measures appear to be of the same order by both methods.

There is an important difference, however, where the filar micrometer scores over the comparison image micrometer. On a night of very bad definition, when the diffraction disks are never visible, the components of a wide pair may be so far apart that the two disks of diffusion are quite separate. When this is the case the artificial star can never be made to match the real star and the observer in attempting to match the distance between two small points of light with that between two diffused patches is liable to measure the separation too small. With the filar micrometer a reliable distance observation is still possible under these circumstances, as the two diffused images can be bisected with wires almost as accurately as the smaller diffraction disks.

Close Pairs.—It is on the close pairs (< 1 second separation) where the great advantages of the new method are most apparent, and we have obtained some remarkable results.

The seeing at Greenwich is such that on most nights the images are large diffused disks with the diffraction disk appearing as a nucleus in glimpses of very short duration. On a close pair these disks of diffusion overlap, so that the double star appears as an elongated patch with glimpses, of about half-a-second duration, of two nuclei. These glimpses are more frequent the better the seeing. With average seeing they may appear at intervals of about ten seconds or so.

Under these conditions it is impossible to measure the pair with the filar micrometer. Each time the observer attempts to place the wires on the nuclei they disappear. With the comparison image micrometer, however, it has been found that these glimpses of the nuclei are of quite long enough duration to observe whether the real and artificial images match. If an adjustment is necessary it is made and the next glimpse shows whether the match is satisfactory.

As the nuclei flash out the strong resemblance between the artificial and real images is such as to give the observer confidence that an error of 10 per cent. in the estimated separation would be obvious.

The increase in the number of observations of close pairs which this condition makes possible is indicated already by comparison of measures obtained in the past year with the filar micrometer with those obtained since the comparison image has been in use. During the whole of last year only 19 measures of separation less than half a second were obtained,

whereas 16 have been made between October 8 and December 15. These were obtained on 9 nights and there was never an occasion when the filar micrometer could have been used on these close stars.

The Measures.—The results obtained on 20 pairs are given, together with measures made recently by Baize, Simonow, van Biesbroeck and Greenwich with the filar micrometer.

It will be noticed that for pairs with separation greater than a second the comparison image micrometer is in agreement with the filar micrometer measures, but for the closer pairs there are systematic differences in the observed separations. When there is a comparison with Baize or with Greenwich filar micrometer measures the new micrometer in some cases gives a smaller separation, but in every case where there is a comparison with van Biesbroeck's results or with the ephemeris of a very recently published orbit they agree extremely closely. Baize uses a 12-inch telescope and evidently experiences difficulty in measuring these close pairs, for he states that for stars with separation less than $0''\cdot7$ he replaces the usual bisection of the stars by an estimation of their separation made by placing the wires so that their distance apart appears equal to that of the stars. This is in effect what we are doing with the comparison image micrometer, except for the important difference that the scale which we place beside the real pair is a replica of it. We feel that if two wires are placed beside the pair in an effort to measure the separation in this way there is a danger that the *space between the wires* will be made equal to the star's separation instead of the distance between the centres of the wires. This would account for this systematic difference.

Van Biesbroeck, on the other hand, is using a 40-inch telescope of long focal length, and an inspection of his measures indicates that he can measure stars as close as $0''\cdot15$ with comparative ease, in which case the pairs considered here should be quite easy objects.

In view of the extraordinary resemblance between the real and artificial star images on these close pairs we feel that the measures of separation cannot possibly be much in error, and in this case the results from last year's observations at Greenwich with the filar micrometer are too large on these stars.

The claim of great accuracy made by Mr. Hargreaves for the method seems fully justified. The filar micrometer is still available, and our future observing policy will probably be to measure the very wide pairs with it, using the comparison image micrometer for the closer pairs and for stars whose faintness causes difficulty with the older method of observing. An interferometer with which it is proposed to measure some of the very close pairs is nearing completion, and with the three observing methods simultaneously available the 28-inch telescope should prove a powerful instrument for double-star work.

[TABLE

Comparison Image Micrometer Measures			Filar Micrometer Measures			
ADS 746. OΣ 20. 5·9–7·0. Ao.						
				°	″	
			1931·69	281·2	0·48	Simonow
			31·93	287·4	0·63	,,
			32·05	288·5	0·38	,,
			34·81	284·7	—	,,
			1932·62	285·5	0·50	,,
	°	″				
1937·79	276·5	0·48	1933·02	283·7	0·76	Baize
37·84	279·5	0·48	33·91	285·4	0·59	,,
37·88	274·2	0·49	34·11	286·5	0·49	,,
37·90	275·4	0·51	34·12	282·3	0·59	,,
			35·05	282·4	0·45	,,
1937·85	276·4	0·49	1934·04	284·1	0·58	,,
			1937·03	286·0	0·54	Gr. O. (RW)
			From orbit by Zagar (1926) :—			
				°	″	
			1932·0	284·8	0·42	
			34·0	279·5	0·35	
			36·0	272·5	0·26	
			38·0	255·7	—	

The pair is not closing in as rapidly as foreseen by Zagar and periastron passage will probably occur later than 1939.

Comparison Image Micrometer Measures			Filar Micrometer Measures			
ADS 755. Σ 73. (36 Andromedæ.) 6·2–6·8. Ko.						
				°	″	
			1931·69	86·7	0·56	Simonow
			31·75	83·9	0·75	,,
			31·76	83·5	0·85	,,
			31·85	85·0	0·53	,,
			31·91	86·1	0·50	,,
			31·93	91·2	0·71	,,
			1931·82	86·1	0·65	,,
	°	″				
1937·79	108·5	0·55	1933·08	92·8	0·66	3*n* Baize
37·84	111·8	0·51	34·12	97·3	0·60	4 ,,
37·86	108·3	0·53	35·05	99·8	0·57	5 ,,
37·88	104·7	0·51				
37·90	109·8	0·55	1936·86	102·8	0·75	Gr. O. (LS)
			36·98	105·3	0·68	,, (LS)
			37·00	108·1	0·71	,, (RW)
			37·04	105·6	0·71	,, (LS)
1937·85	108·6	0·53	1936·97	105·4	0·71	,,
			From Berman's orbit (1926) :—			
				°	″	
			1931·0	88·4	0·52	
			33·0	101·7	0·43	
			35·0	123·0	0·32	
			37·0	169·4	0·20	

An approximate extrapolation of the results given in Aitken's catalogue gives the separation at the mean date of Simonow's, Baize's and the comparison image micrometer observations as 0″·63, 0″·60 and 0″·54 respectively. The discordance, therefore, is merely apparent and can be accounted for by the orbital motion between the dates. The Greenwich filar micrometer results appear to be too large. Evidently the pair is not closing in as rapidly as indicated by Berman's orbit.

Comparison Image Micrometer Measures			Filar Micrometer Measures			
ADS 955. β 303. 7·1–7·3 Fo.						
			1933·91	288·7	0·74	Baize
			34·06	289·3	0·64	,,
			34·12	285·9	0·66	,,
	°	″				
1937·79	287·0	0·58	1934·03	288·0	0·68	,,
37·84	290·9	0·56				
37·86	288·4	0·59	1936·84	287·6	0·68	Gr. O. (RW)
37·88	285·5	0·63	36·92	289·5	(0·84)	,, (RW)
37·90	285·1	0·59	36·98	287·2	0·66	,, (LS)
			37·00	287·9	(0·78)	,, (RW)
1937·85	287·4	0·59	37·03	286·1	0·65	,, (RW)
			1936·95	287·7	0·66	,,
ADS 1005. Hu 803. 8·5–9·5. —.						
			1928·76	181·8	0·59	van Biesbroeck
			28·83	181·7	0·54	,,
	°	″	29·85	179·9	0·65	,,
1937·79	181·0	0·61	32·75	183·9	0·53	,,
37·90	184·8	0·55	32·85	183·5	0·58	,,
37·90	183·0	0·60				
1937·86	182·9	0·59	1930·61	182·2	0·58	,,
ADS 1473. Ho 311. 7·0–7·2. A5.						
	°	″				
1937·79	190·5	0·45				
37·90	188·2	0·45	1936·98	190·9	0·64	Gr. O. (LS)
1937·85	189·4	0·45				
ADS 1579. Σ 194. 8·0–8·3. A3.						
			1933·91	273·7	1·28	Baize
			34·04	274·1	1·30	,,
			34·06	272·9	1·13	,,
	°	″				
1937·79	273·6	1·22	1934·00	273·6	1·24	,,
37·88	273·2	1·21				
37·90	271·7	1·15	1936·86	271·5	1·16	Gr. O. (LS)
			36·89	272·4	1·25	,, (RW)
1937·86	272·8	1·19	36·96	273·9	1·28	,, (N)
			36·97	271·7	1·35	,, (N)
			37·00	273·5	1·22	,, (RW)
			1936·94	272·6	1·25	,,

This pair is opening out very slowly. The agreement between the comparison image micrometer and van Biesbroeck is perfect.

Comparison Image Micrometer Measures			Filar Micrometer Measures			
ADS 1630. OΣ 38. (γ^2 Andromedæ.) 5·0–6·2. Ao.						
				°	″	
			1931·72	105·4	0·47	4*n* Simonow
			32·78	105·3	0·59	3 ,,
			1933·77	103·9	0·45	3*n* van Biesbroeck
			1933·09	106·0	0·49	4*n* Baize
1937·90	103·5	0·46	34·12	103·1	0·47	4 ,,
			35·14	103·4	0·45	3 ,,
			1937·07	104·7	0·48	3*n* Gr. O. (RW)
			From orbit by Woolley and Symms :—			
				°	″	
			1937·9	103·5	0·40	
ADS 2616. Σ 412. (7 Tauri.) 6·6–6·7. A2.						
				°	″	
			1925·42	80·5	0·26	4*n* van Biesbroeck
			26·92	77·3	0·26	3 ,,
			29·87	64·2	0·29	4 ,,
			34·14	50·0	0·35	3 ,,
			1924·91	81·9	0·26	4*n* Van den Bos
	°	″				
1937·90	47·6	0·34	1926·05	59·3	0·37	Gr. O. (HF)
37·95	50·7	0·34	34·07	47·7	0·34	,, (HF)
			37·14	45·4	0·39	,, (RW)
1937·93	49·2	0·34	37·15	45·4	0·33	,, (RW)
			From orbit by Luyten (1934) :—			
				°	″	
			1925·0	75·3	0·22	
			30·0	60·1	0·26	
			35·0	48·8	0·30	
			40·0	40·2	0·34	

The separations observed with the comparison image micrometer agree with van Biesbroeck and with the orbit by Luyten. It may be noted that while making the observation of date 1937·15, given under Gr. O. filar micrometer results, the observer remarked on the very excellent seeing. Evidently, given perfect conditions, the 28-inch filar micrometer results agree with van Biesbroeck and the comparison image micrometer. Two of the earlier Gr. O. separations appear large.

Comparison Image Micrometer Measures			Filar Micrometer Measures			
ADS 3082. OΣ 77. 7·5–7·5. F8.						
				°	″	
			1931·88	243·9	0·62	7*n* Simonow
			35·22	249·0	0·63	1 ,,
			1933·22	247·5	0·63	2*n* Baize
			35·22	246·7	0·82	2 ,,
	°	″				
1937·90	247·2	0·62	1930·81	245·3	0·68	6*n* van Biesbroeck
			34·85	244·8	0·65	4 ,,
			1936·95	246·7	0·70	4*n* Gr. O. (RW)
			From orbit by Hynek (1935):—			
				°	″	
			1934·0	246·9	0·62	
			36·0	248·6	0·63	
			38·0	250·2	0·64	
ADS 11005. Σ 2262. (τ Ophiuchi.) 5·0–5·7. Fo.						
	°	″		°	″	
1937·73	266·3	1·99	1931·54	264·7	1·77	4*n* Simonow
37·76	266·3	2·03	32·47	265·2	2·04	4 ,,
			1934·66	264·2	1·92	3*n* Baize
1937·75	266·3	2·01				
			1936·53	267·1	2·12	6*n* Gr. O.
ADS 11635. Σ 2382. (ϵ^1 Lyræ.) 4·6–6·3. A3.						
				°	″	
			1933·92	4·8	2·82	4*n* Baize
	°	″				
1937·75	6·7	2·93	1936·68	2·4	2·98	Gr. O. (N)
37·76	3·5	2·92	36·73	4·5	2·91	,, (N)
			36·76	6·8	2·97	,, (RW)
1937·76	5·1	2·93	1936·72	4·6	2·95	,,
Σ 2383. (ϵ^2 Lyræ.) 4·9–5·2. A5.						
				°	″	
			1933·92	111·8	2·33	4*n* Baize
	°	″				
1937·75	112·1	2·23	1936·68	111·5	2·42	Gr. O. (N)
37·76	109·5	2·14	36·73	111·8	2·26	,, (N)
			36·76	111·6	2·31	,, (RW)
1937·76	110·8	2·19	1936·72	111·6	2·33	,,
ADS 11811. β 137. 8·2–8·7. Go.						
	°	″		°	″	
1937·75	139·6	1·37	1933·74	142·0	1·31	3*n* Baize
			36·63	141·7	1·40	5*n* Gr. O.

Comparison Image Micrometer Measures			Filar Micrometer Measures			
ADS 11956. Σ 2437. 7·8–8·0. G5.						
	°	″		°	″	
1937·74	40·6	0·66				
37·79	51·3	0·58	1935·46	46·8	0·80	4*n* Baize
37·80	47·9	0·65				
37·80	47·5	0·59	1936·55	47·7	0·85	5*n* Gr. O.
1937·78	46·8	0·62				
ADS 12040. Σ 2454. 8·0–9·2. Ko.						
	°	″		°	″	
1937·74	265·9	0·77	1932·62	264·0	1·10	5*n* Simonow
37·75	267·0	0·86	34·99	266·4	1·02	4*n* Baize
			37·06	266·1	0·96	3*n* Gr. O.
1937·75	266·5	0·82				
ADS 12067. J 813. 9·4–9·8. —.						
	°	″				
1937·74	2·7	1·00				
ADS 12972. OΣ 387. 7·2–8·2. F5.						
				°	″	
			1931·66	282·6	0·50	4*n* Simonow
			32·63	282·4	0·54	4 ,,
			1933·75	282·6	0·53	2*n* Baize
			34·69	279·0	0·50	2 ,,
			35·78	274·7	0·50	4 ,,
	°	″				
1937·77	272·3	0·42	1925·49	301·1	0·50	3*n* van Biesbroeck
37·79	271·4	0·40	26·50	299·4	0·49	2 ,,
37·80	273·2	0·42	29·50	291·2	0·47	2 ,,
			35·57	276·3	0·40	3 ,,
1937·79	272·3	0·41				
			1936·65	274·2	0·55 ±	4*n* Duruy
			1936·89	271·8	0·52	4*n* Gr. O.
			Voronov's orbit (1934) gives :—			
				°	″	
			1933·6	267·4	0·37	
			35·7	255·7	0·34	
			37·8	242·5	0·33	

This pair is closing in, but not as rapidly as Voronov's orbit foresees. Allowing for orbital motion, the separations measured by the comparison image micrometer accord with van Biesbroeck's.

Comparison Image Micrometer Measures			Filar Micrometer Measures			
ADS 13277. OΣ 395. (16 Vulpeculi.) 5·8–6·2. Fo.						
	°	″		°	″	
1937·79	109·5	0·76	1931·63	108·7	0·67	6*n* Simonow
37·80	109·3	0·70	32·53	109·1	0·80	4 ,,
37·80	110·9	0·73	1934·49	109·4	0·85	4*n* Baize
1937·80	109·9	0·73	1936·59	110·4	0·91	5*n* Gr. O.
ADS 16836. β 720. (72 Pegasi.) 6·0–6·0. K2.						
	°	″		°	″	
1937·79	224·5	0·35	1932·82	209·0	0·40	2*n* Simonow
37·88	223·4	0·35				
37·90	212·2	0·35	1934·44	221·3	0·42	4*n* Baize
37·94	219·2	0·35				
37·95	218·1	0·35	1934·69	215·3	0·43	1*n* van Biesbroeck
			35·77	215·7	0·36	1 ,,
1937·89	219·5	0·35				
ADS 16951. A 1242. 9·0–9·0. G5.						
	°	″		°	″	
1937·80	295·9	0·65	1928·57	291·5	0·60	7*n* van Biesbroeck
37·88	289·8	0·62	31·92	292·9	0·57	5 ,,
1937·84	292·9	0·63	1933·73	292·6	0·62	2*n* Aitken

This pair has been used as a test object to estimate the seeing; this accounts for the large number of observations. The agreement of the separation measures is remarkable. There was never an occasion when it was possible to measure the pair with the filar micrometer.

Summary.—The paper deals with a method of measuring double stars by comparison with an artificial double star.

Part I describes the instrument, in which a Wollaston prism is used to produce the double star, the relative brightness of the components being controlled by means of crossed Nicols.

Part II describes the method of use. Some results of measures of double stars made with the micrometer are given, and compared with filar micrometer results.

Note added 1938 *February* 5.—The following observations of three of the above stars * were made by Mr. F. J. Hargreaves, using his original comparison image micrometer mounted on his 12-inch reflector, focal length 78 inches, at the Newtonian focus. The agreement between these measures

* *B.A.A. Handbook*, 1933 and 1934.

and those made with the Greenwich instrument (allowing in the case of the first star for orbital motion) is striking.

					°	″
ADS	755	Σ	73	1931·8	88·6	0·60
				1932·9	93·2	0·57
ADS	1630	OΣ	38	1932·0	102·7	0·54
				1932·9	105·3	0·52
ADS	13277	OΣ	395	1931·7	108·8	0·80

Royal Observatory,
Greenwich:
1938 *January* 7.

THE CORRECTION OF FREQUENCIES FOR A KNOWN STANDARD ERROR OF OBSERVATION.

Harold Jeffreys, D.Sc., F.R.S.

Eddington has discussed the following problem.* A series of quantities are measured and classified in equal ranges. Each measure has a known uncertainty. On account of the errors of measurement some quantities are put into the wrong ranges. If the true number in a range is greater than those in the adjacent ranges, we should expect more observations to be scattered out of the range than into it, so that the observed number will need a positive correction. The problem is to estimate this correction. Several astronomers have discussed the application of the method with me, having found that its tendency is to increase the irregularity of the data. It appears that this might have been expected, and that the method is fallacious and would in any ordinary case increase the errors.

The quantities considered being star magnitudes, Eddington proceeds by introducing a "true number" $v(m)dm$ between magnitudes m and $m+dm$, and an "observed number" $u(m)dm$. He gives an integral equation expressing $u(m)$ in terms of $v(m)$, namely,

$$u(m) = \frac{h}{\sqrt{\pi}} \int_{-\infty}^{\infty} v(m+x) \exp(-h^2x^2) dx, \qquad (1)$$

where h is the precision constant of the observations. He solves this operationally and obtains

$$v(m) = \exp\left(-\frac{1}{4h^2}\frac{d^2}{dm^2}\right) u(m). \qquad (2)$$

The derivatives are then expressed in terms of differences and an expression for $v(m)$ is obtained.

Several criticisms of this procedure may be offered. Since dm is a differential, the number of stars, either actual or observed, in a range dm is either 0 or 1, usually 0. $u(m)$ and $v(m)$ are either 0 or ∞. Consequently (1) is meaningless in terms of ordinary integration. It could be given a meaning in terms of a Stieltjes integral by introducing a function $V(m)$ equal to the total number of stars with magnitudes less than m, and writing

$$u(m) = \frac{h}{\sqrt{\pi}} \int \exp(-h^2x^2) dV(m+x) = \frac{h}{\sqrt{\pi}} \int \exp\{-h^2(m-n)^2\} dV(n) \qquad (3)$$

in which the ranges where $v(n) = 0$ and therefore $V(n)$ is constant contribute nothing, but wherever $V(n)$ jumps by 1 the term $\exp\{-h^2(m-n)^2\}$ is added to the integral. The result would still be a continuous $u(m)$; the essential

* *M.N.*, 73, 359–360, 1913. Reproduced in Brunt's *Combination of Observations*, p. 53, 1931.

feature that the observed values also form a discrete series is not represented by (3). To get the actual distribution of the data we should need to know, not merely the law of error, but every individual error, and this by hypothesis is not known. Further, if $u(m)$ had its actual properties the operation expressed in (2) would be meaningless. (Expansions in ascending powers of the differential operator should always be mistrusted until they have independent justification. All of Heaviside's difficulties are due to such expansions.)

It is difficult to believe that Eddington really meant that $u(m)$ and $v(m)$ have the above properties, in spite of the perfectly clear statements implied in the definitions. If the analysis is to mean anything they must be replaced in some way by continuous functions, and $u(m)dm$ and $v(m)dm$ cease to be the observed and true numbers. This can be done if we regard $u(m)$ and $v(m)$ as probability densities. We can imagine a distribution of the chance that a star with other assigned properties will lie in a particular range of magnitude, and since there is no reason to suppose that the possible magnitudes of stars are limited to a set of discrete values, already specified with mathematical accuracy, the chance that the true magnitude of a star will lie in a range dm may well be expressed by a continuous function $v(m)$. In that case the chance that the observed magnitude will lie in a particular range dm is given correctly by (1), by the addition rule for probabilities. If N is the total number of stars observed, $N\int u(m)dm$ through any finite range will be the expectation of the number observed in that range, given N and the supposed law of the distribution of chance. This is not, however, the actual observed number in the range. If it was, there would be no error of sampling, and this would be inconsistent with the stars being a random sample from the class that they are intended to represent. The observed number will differ from the expectation by a random error, which is represented approximately by a standard error equal to the square root of the expectation. Thus neither $u(m)$ nor the expectation of the observed number x in any range is known; they are unknown quantities, which we are trying to estimate from the actual observed numbers, and any estimate of them will have an uncertainty, which will be transmitted to $v(m)$ by the method of analysis.

It can be seen at once that the inclusion of the second term in Eddington's series, if applied to the observed numbers in the ranges, will increase the random error. For if we have a series of observed quantities at equal intervals of an argument we can smooth them considerably and at the same time reduce the random error by adding to each $\frac{1}{3}$ or $\frac{1}{4}$ of the second difference centred on it. But Eddington's correction is a negative multiple of the second difference, and must increase the random error. Admittedly, if there is a genuine second derivative that can be determined significantly from the data the simple smoothing process will introduce a systematic error proportional to it. The conditions of the present problem require that there is already a systematic error, since the tendency of the observational error is to smooth out genuine irregularities, and the object of the correction is to remove this effect. But it remains to be seen whether the increase of the random error is not enough to compensate for this benefit.

The danger can be seen in another way from an analogy with a problem of heat conduction in a long rod. If $v(m)$ represents the initial distribution of temperature, the distribution at time t has the form (2), h^2 being replaced by $1/4kt$, where k is the thermometric conductivity. The problem of inferring $v(m)$ from $u(m)$ is therefore mathematically identical with that of finding the initial distribution of temperature from that at a later time. It is well known that this cannot be solved in general. If $d^2u(m)/dm^2$ does not exist at any point the solution cannot be taken back at all. If $u(m)$ is an actual distribution of temperature, it will be possible to solve the problem of the distribution at earlier times, provided that these are later than the actual initial times ; but if for any time $d^2v(m)/dm^2$ did not exist at a point the solution could be taken back no further. The failure of the solution will be shown by the divergence of the series.

Now in the statistical problem $u(m)$ is a genuine chance, and if it was known we should have a case where the solution for $v(m)$ could be carried out. But we do not know $u(m)$; we know only the observed numbers over finite ranges of m. If the ranges are short, so that the numbers in them have an appreciable random error, this introduces just the kind of irregularity that will prevent the solution from being taken back at all. Comparing (2) with the thermal analogue we see that it would be equivalent to an expansion of the temperature in positive powers of the time. If the process was valid the inclusion of more terms of the series would be equivalent, roughly, to making the solution applicable further back in time. But if the irregularities of temperature did not increase as we go further back in time we should have a contradiction of the second law of thermodynamics.

If the observed totals by ranges are x_r, and the corrected ones, retaining the second term of the solution, are y_r, we have

$$y_r = x_r - \frac{1}{4h^2a^2}\delta^2 x_r \tag{4}$$

$$= -\frac{1}{4h^2a^2}(x_{r-1} + x_{r+1}) + \left(1 + \frac{1}{2h^2a^2}\right)x_r, \tag{5}$$

where a is the tabular interval. If we use $\sigma(x)$ to denote the standard random error of x, we have therefore

$$\sigma(y_r) > \left(1 + \frac{1}{2h^2a^2}\right)\sigma(x_r) \tag{6}$$

and the first factor is always greater than 1. Thus the inclusion of the first term of the correction always increases the random error.

It will be correct to say that

$$v_r = u_r - \frac{1}{4h^2a^2}\delta^2 u_r \tag{7}$$

approximately, if by u_r and v_r we mean the (so far unknown) expectations in the ranges, provided that the second term is small compared with the first and that higher differences are small; but y_r will differ from v_r, on the whole,

by more than x_r differs from u_r. It may indeed happen that y_r differs from v_r by more than x_r from v_r. A necessary condition that there should be a systematic improvement is

$$\sigma^2(x_r - v_r) = \sigma^2(x_r - u_r) + (v_r - u_r)^2 > \sigma^2(y_r - v_r) \tag{8}$$

or

$$\frac{1}{16h^4\alpha^4}(\delta^2 u_r)^2 > \frac{1}{h^2\alpha^2}\sigma^2(x_r) = \frac{u_r}{h^2\alpha^2}. \tag{9}$$

For given spacing, the left side increases with the number of observations, N, like N^2, the right like N. For given N, the left side depends little on the spacing if it is already short compared with the distances between successive maxima of u_r, while the right is proportional to $1/\alpha$. Hence large N and large α favour the utility of the correction, provided that we have enough observations to obtain a significant estimate of $\delta^2 u_r$. The standard error of x_r shows that the condition required is that

$$|\, \delta^2 x_r \,| >> \sigma(x_{r-1} - 2x_r + x_{r+1}) = \sqrt{(6x_r)} \tag{10}$$

nearly. We cannot replace $\delta^2 u_r$ by $\delta^2 x_r$ in (7) unless this is satisfied. If we can, (9) becomes

$$(\delta^2 x_r)^2 > 16h^2\alpha^2 x_r. \tag{11}$$

The remarkable thing about this condition is that the right side increases with $h\alpha$; that is, the more of the observed values may be expected to have been rightly classified to begin with, the harder it is to satisfy the condition. The fact that large $h\alpha$, and therefore accurate observing, favour the smallness of the correcting terms in (2) suggests the convergence of the series; but unfortunately it is just then that the correction is most likely to make matters worse. On the other hand, if $h\alpha$ is small enough to satisfy (11), the convergence of the series, problematical in any case, is made much slower.

These restrictions are so severe that I doubt whether the method could ever be correctly applied in practice, and in any case better methods exist. Before trying to correct for the error of measurement, which is presumably always a small part of the whole scatter of the observations, it would be better to try to reduce the sampling errors by proceeding at once to the determination of a smooth form for u_r from the data x_r. If it should turn out that the x_r are consistent with the normal distribution for $u(m)$ with standard variation s, as judged by the χ^2 test, the rest is easy; for the distribution for $v(m)$ will be a normal one with standard variation $(s^2 - 1/2h^2)^{\frac{1}{2}}$ and the same mean. For more complicated laws it would still be possible to find the parameters by the method of maximum likelihood of Fisher; * we could either proceed directly to $v(m)$ or infer it from $u(m)$ by (7). Even when no apparent formal law will fit the data satisfactorily, it will be possible to find what I have called "summary values" for u_r over several intervals,†

* *Statistical Methods for Research Workers*, 1936, and references there given.

† *Proc. Camb. Phil. Soc.*, **33**, 444–450, 1937; *M.N., Geophys. Suppl.*, **4**, 174, 1937.

treating the x_r as data with standard errors $\sqrt{x_r}$, and to derive u_r by interpolation by divided differences. Then v_r can be found as before, provided the $\delta^2 u_r$ as estimated are larger than their standard errors. If they are not, it will be better to leave u_r unaltered.

In this discussion $v(m)$ and $u(m)$ have been taken as chances. This is necessary if the results are to be supposed to have any relevance beyond the original data, which must be regarded as a sample from a larger class whose properties we are trying to estimate. It is not necessary that this class should be infinite, as many statistical writers suppose; this device makes ratios illusory, and the sufficient justification of the notion of chance * is that it is a good approximation to an accurate probability treatment when the class sampled is large compared with the sample. If the problem is merely to say how many of the stars actually observed have been correctly placed, a formal solution is possible; but I doubt whether this is what is really wanted. It will, however, be in terms of probability even in this case. If any method appears to lead to an exact solution it would amount to saying that we know the individual errors, and it must necessarily be wrong. Eddington remarks at the end of his paper that if there are only a finite number of groups, as will normally happen, the series of differences will terminate, the data can be replaced by a polynomial, and convergence will be satisfied; but in that case the answer will be unique and therefore must be wrong.

Summary.—A method given by Eddington for the correction of frequencies for a known uncertainty of observation will in practice increase the random error of sampling, and this effect will often be greater than the improvement given by the allowance for observational error, particularly when the standard error of observation is small compared with the grouping interval. It appears that this will account for the difficulties that have been found in the application of the method. A better way is to begin by applying one of the usual methods of fitting distributions of chance to the observed frequencies and, if necessary, to apply Eddington's method only after the random error has been reduced in this way.

* That is, of a probability of an observation, given a law, that is expressed completely by the parameters in that law, and given these is independent of previous observations.

THE RED SHIFT OF THE D LINES OF SODIUM IN THE SUN.

J. Evershed, F.R.S.

In the *Observatory* for 1923 October (**46**, 303) I have recorded the results of some measures of the shifts of the D lines of sodium at the centre of the Sun's disc and at the limb. These were obtained with the large grating spectrograph of the Kodaikanal Observatory, using a 45-mm. image of the Sun and the sharp sodium emission lines given by a carbon-iron arc containing some crystals of a potassium salt.

The results, corrected for the air pressure at Kodaikanal and assuming zero pressure in the Sun, indicated shifts towards red slightly in excess of the Einstein relativity effect, both at the centre of the Sun and at the limb. Unlike the lines of iron, there was no increase of wave-length in passing from centre to limb. At both positions the mean value of D1 and D2 was +·015 A. But this value depends very largely on the reliability of measures of pressure shift. According to Humphreys (*Astrophysical Journal*, **6**, 210), the pressure shift for 1 atmosphere is approximately +·011 A. both for D1 and D2.

During the year 1937 I have redetermined the shift for the D1 line only, using the liquid prism spectrograph with ethyl cinnamate or methyl naphthalene as the dispersing agent. This last has a slightly greater dispersive power than cinnamate; with 10 transmissions through the prism it gives nearly double the linear dispersion of the grating plates, with about half the exposure time on the Sun.

The comparison spectrum is produced by a low-pressure sodium lamp made by Messrs. Philips. This lamp contains solid sodium metal and neon gas at a pressure not exceeding 4 cm. of mercury at room temperature. When the electric current is first applied the red light of neon appears, but as the tube warms up sodium evaporates, and after five minutes fills the tube with a dazzling yellow light. The neon spectrum then is replaced by the D lines of sodium. At the working temperature of about 270° C. the pressure in the tube would be raised to about 1/10th atmosphere. The D lines are remarkable for their sharpness and brilliance, never exceeding in width the solar absorption lines, and of an intensity comparable with the continuous spectrum near D of the Sun itself. This has the great advantage that equal and simultaneous exposures can be made on Sun and lamp. With 10 transmissions through the prism an exposure of 30 seconds is sufficient for process panchromatic plates.

The lamp is placed near the slit of the spectrograph and to one side of it, the light entering the slit through two small reflecting prisms. The Sun's image, 60 mm. in diameter, falls directly on the slit, and any part of it, centre or limb, is directed between the two prisms, which are separated by an interval

of 2 mm. The collimator-camera lens of the Littrow spectrograph is completely filled with light from both sources.

The linear dispersion obtained with 10 transmissions through methyl naphthalene in a 63° prism and a lens of 16 feet focus is ·69 A. per mm. at D1, the two sodium lines being separated by nearly 9 mm. D2 in the moist climate of England cannot be used in this research, owing to a water-vapour line which broadens it on the red side, more or less according to the humidity of the air, the altitude of the Sun, and the relative motion of Earth and Sun. It is seen separated from D2 only at the Sun's east limb in an autumn morning, when the solar line is shifted nearly the maximum amount towards blue.

Confining attention therefore to D1, the following results have been obtained, using the positive on negative method of measuring, for which the spectra are well adapted.

Red Shift of the Sodium Line D1.

No. of Spectra measured	Position	Sun – Sodium Tube	
12	Centre of disc	+·0123 ± ·0003	
12 pairs	Equator	+·0115	Mean of limb spectra +·0123 ± ·0002
22 pairs	12° to 24°	+·0130	
10	North pole	+·0120	
10	South pole	+·0120	

The spectra of the Sun's equator and between latitude 12° and 24° were taken in pairs, east limb and west limb, the shift being derived from half the difference W.–E. to eliminate the solar rotation effect.

The differences here shown at different positions on the limb are probably of no significance, and the mean of 54 estimates from all the limb spectra agrees with the centre shift.

These are the observed values of the shifts, corrected for the orbital and diurnal motions of the Earth relative to the Sun, but not corrected for difference of pressure in the two sources. Assuming a zero pressure in the higher parts of the reversing layer, represented by the D line, and a pressure of 1/10th atmosphere in the sodium lamp, then, from Humphreys' measures of the pressure shift of D1, ·0011 A. must be added to the figures given above, and the mean shift then becomes +·0134 A., both at centre and limb, the relativity shift equivalent to ·634 km./sec. being +·0125 A.

Whether this small correction for pressure is applied or not, it is evident that D1 gives a close approximation to the theoretical shift both at the centre and limb, as had previously been found from 33 grating spectra of less dispersion but in which D2 was included.

The sodium lines therefore differ from the iron lines, which give shifts at the limb nearly twice the relativity effect, and a marked increase of shift between centre and limb for the lines representing lowest levels. They differ also from the calcium lines H3, K3, which give the same displacements as iron at the limb, but a change of wave-length between centre and limb in the opposite sense, the redward shift increasing from limb towards centre, where it becomes actually three times the relativity effect.

These changes of wave-length between centre and limb are most readily explained by movements of the solar gases radially outward in the case of low-level iron, and inward (that is, a motion of descent) for calcium in the higher chromosphere.

It is true that such measures as have been made of low-level lines at intermediate points between centre and limb give a law of increase of wave-length differing from the simple theoretical relation on the assumption of uniform radial streaming ; but this I think is to be expected from the fact that near the limb the absorption lines represent a higher level than at the centre, and the velocity of outflow probably accelerates, as it does in the case of eruptive prominences and in the outflow from sunspots.

The constancy of the limb shift, or of the difference of wave-length between centre and limb, has been called in question by A. Hunter (*M.N.*, **94**, 600, 1934) from the results of measures of two solar rotation plates from the Edinburgh Observatory obtained in the year 1932. These gave much smaller values limb – centre than had been observed in previous years, especially by Halm in 1907.

It is uncertain whether this apparent change is due to a temporary variation of wave-length at the centre of the disc or to a secular change of wave-length at the limb, as Hunter appears to assume. My own experience is that temporary changes of the shift limb – arc or limb – centre frequently occur, but innumerable measures of limb shift over a series of years give no certain evidence of long-period changes. To test this possibility, however, four pairs of east and west limb spectra in the H and K region were obtained in November last, and these were found to yield limb – arc shifts for the iron lines in close agreement with previous values—that is, nearly twice the Einstein prediction. It cannot be supposed therefore that the smaller limb shift of the sodium line D1 indicates a general change.

Sodium, and probably also aluminium, represented by the two lines of that element between H and K, occupies an intermediate position between the calcium of the higher chromosphere and the low-level iron, a position where there appears to be no radial movement outward or inward. A similar intermediate position is taken by sodium in sunspots, where low-level iron lines show outward motion and high-level calcium lines inward motion, the D lines of sodium being undisplaced. But in this case the motion is parallel to the Sun's surface, not radial to the Sun. It is as if the normal radial outflow at low levels and inflow at high levels were increased in sunspots and changed in direction from vertical to horizontal, the intermediate zone represented by the D lines remaining undisturbed.

But in whatever way we interpret the changes of wave-length across the disc of the Sun, or in sunspots, and the large shift of the iron and calcium lines at the limb, amounting to nearly twice the predicted shift, it is evident that the sodium lines do not share in these changes, and a factor of 2 is not required to bring theory into line with observation ; for the measures herein recorded indicate a close approximation to the prediction of Einstein, both at the centre of the disc and the limb.

I am very glad to acknowledge here the assistance given to me by

Mr. Hargreaves in the essential part he has played in perfecting the performance of the liquid prism, and for the loan of the sodium tube and its accessories.

Summary.—By means of multiple transmission through a liquid prism containing methyl naphthalene, measures have been made of the shift of the line D1 in high-dispersion spectra of the Sun compared with similar spectra of a low-pressure sodium lamp. A table is given showing the shifts, corrected for the motion of the Earth, at the centre of the Sun, at the equatorial and polar limbs and in spot latitudes. Unlike the lines of iron or calcium, the sodium line is found to give the same shift towards red at the centre of the disc as at the various points on the limb, and this shift agrees very closely with the Einstein relativity effect. The constancy of the shift of the solar lines was tested by photographing the iron lines in the H and K region, and these spectra were found to yield the same values for the shifts as had been observed in former years, namely, nearly twice the relativity effect, at the Sun's limb.

FITTING ATMOSPHERES TO STARS.

Henry Norris Russell.

There has been considerable recent discussion of the effect which would be produced upon a star if its outer layers (comprising but a very small portion of the whole mass) were replaced by material of different opacity. It can be shown without difficulty that the effect upon the characteristics of the star as a whole—luminosity, radius, and the like—will be small.

If the star is in equilibrium—mechanical, radiative, and secular (that is, with respect to energy-generation)—its internal constitution will be determined by the familiar equations (in an equally familiar notation):

$$\begin{aligned} \frac{dP}{dr} &= -\frac{GM(r)}{r^2}\rho, & M(r) &= \int_0^r 4\pi r^2 \rho dr, \\ \frac{dp'}{dr} &= -\frac{KL(r)}{4\pi c r^2}\rho, & L(r) &= \int_0^r 4\pi r^2 \epsilon \rho dr, \end{aligned} \tag{1}$$

where P is the total pressure and p' the radiation pressure. The density ρ, the opacity K, and the rate of energy generation ϵ will in general be functions of P, p', and of the composition of the material.

Let us first suppose that the composition is uniform, so that ρ, K, ϵ are functions of P and p' alone. It is well known that the equations (1) may be solved by quadratures, starting at the centre with a fixed value of P_c and various values of p_c', and carried outward till one is found which satisfies the boundary conditions that, as $r \to r_1$, $P - p' \to 0$, $p' \to \frac{1}{6}aT_1^4 = L/6\pi c r_1^2$.* Repetition for different initial values of P gives, in principle, the complete set of possible stars of the given composition. These form a one-parameter family, and if the total mass M is specified, the values of L, r_1, T_1, etc. are thereby determined. (Vogt's theorem.)

Quadratures starting with a given P_c but with p_c' slightly different from that which gives the acceptable solution will lead to results which differ very little from the latter except near the boundary, where, for the same value of P, p' will be too small or too great, till the solution "runs wild" close to the surface.

In this region the values of M, L, r_1 will be so nearly equal to the boundary values that we may replace them by these constants, without serious loss of accuracy, so that equations (1) take the simple form

$$\begin{aligned} \frac{dP}{dr} &= -g\rho, & g &= GM/r_1^2, \\ \frac{dp'}{dr} &= -\frac{H}{c}K\rho, & H &= L/4\pi r_1^2, \end{aligned} \tag{2}$$

where g and H are constants.

* E. A. Milne, *M.N.*, 97, 67, 1937.

Now suppose that the outermost layers of the star, of mass m, very small compared with M, are removed and replaced by an equal mass of gas of some other kind, with different molecular weight and opacity, and (for the moment only) that the main inner portion of the star is held unaltered. The total pressure at the bottom of the new layer will be, to the present approximation, $\frac{gm}{4\pi r_1^2}$—the same as before. The constitution of the layer may be determined by integrating (2) downwards, with the boundary conditions $P = p' = L/6\pi c r_1^2$. At the interface, P will go over continuously into the old value for the interior, but there will in general be a discontinuity in p'.

However, there exists a neighbouring solution of equations (1) differing very slightly from the "old" one except near the surface, and hence leading to very nearly the same values of r, $M(r)$ and $L(r)$ for a given value of P, except close to the surface. Here r will be close to r_1, $M(r)$ and $L(r)$ will be substantially equal to M and L; but the value of p' corresponding to a given P will be different. By a suitable choice of such a solution, p' as well as P may be made continuous across the interface, and a composite model obtained in which the conditions of mechanical, radiative and secular equilibrium are everywhere satisfied. The luminosity and radius of this model will differ very little from those of the original. That is, a change in composition of the surface layers will have practically no influence on the general characteristics of the star, which will still be determined, as before, by the properties of the main mass in the interior.

If the mass of the new layer differs from that of the old, but is still negligible compared to M, we have only to choose, among the parallel solutions of (1), the one in which P and p' simultaneously reach the values found for the base of the outer layer, and the argument proceeds as before.

The assumption of uniform internal composition, which has so far been made, may now be discarded in favour of a much more general one, namely, that the star is composed of definite quantities of material of different sorts arranged one above another in concentric shells. The transitions from shell to shell may be abrupt or continuous. In this case the composition depends on $M(r)$ (but not on r itself), and ρ, K, ϵ may be regarded as functions of P, p' and $M(r)$. The solution of the problem by quadratures may be conducted in just the same manner as before, assuming no further change in composition beyond the largest value of $M(r)$ for which one was definitely specified. For any actual star we know that the quadratures, starting with the actual values of P_c and p_c', would "finish off" the solution properly and, in general, they would be accompanied by parallel solutions on each side giving different values of p' for assigned values of P near the surface. The preceding argument may then be applied, and the same conclusion follows.

It may be worth while to illustrate the fitting process by the case: $K = \text{const.}$ Equations (2) then give the very familiar result:

$$p' = \frac{KH}{cg} P + p_0'.$$

In the outer layer

$$p_0' = \tfrac{1}{6} a T_1^4 \left(1 - \frac{KH}{cg}\right)$$

and

$$p' = \frac{K_1 H}{cg} P + p_0'.$$

In the inner layer

$$p' = \frac{K_2 H}{cg} P + p_0''.$$

If P_1 is the total pressure at the interface, the condition of continuity is

$$p_0'' - p_0' = (K_1 - K_2) H P_1 / cg.$$

In general pressure and temperature are continuous across the interface, but the density is greatest on the side where the molecular weight is highest. If this is on top, the equilibrium is unstable against convection. The existing evidence indicates a much greater abundance of hydrogen in the atmospheres of stars than in their interiors—in which case the condition for convective stability is satisfied.

In the foregoing analysis the physically absurd assumption of an abrupt transition from layer to layer has been made, for simplicity of presentation Consideration of the course of the quadratures shows easily that substantially similar results would be obtained in the case of a gradual transition, provided always that the outer layers considered formed a negligible fraction of the whole mass of the star. The writer is therefore led to the following conclusions :—

1. The general characteristics of the stars, such as the relations between mass, luminosity, radius, and effective temperature, are determined by the physical properties of, and processes occurring in, the main mass of a star. Differences in these properties, in the external layers alone, so long as these comprise but a small fraction of the whole mass, will cause only small changes in these general characteristics. In particular, if the main mass is of uniform composition, Vogt's theorem holds true, within small limits of error, whatever the composition of the surface layers.

2. The estimation of the mean abundance of hydrogen in the interior of a star rests upon three propositions :

(*a*) The luminosity of a model star varies enormously with its mass (by 15^m or more).

(*b*) The luminosity varies considerably with the hydrogen content (by some 6^m).

(*c*) The luminosity changes surprisingly little for great changes in the assumed law of energy generation * (by amounts of the order of 1^m).

The first two propositions appear to be firmly established ; the third depends at present upon a rather small number of quadratures and might

* *Cf.* Eddington, *The Internal Constitution of the Stars*, Cambridge (1926), p. 127.

well be tested by further work, especially since Biermann * has shown that the occurrence of zones of convection may make a difference of 2^m in the luminosity.

Unless this effect turns out to be greater in other cases, it should be possible to make an approximate estimate of the abundance of hydrogen in the interior (giving some sort of a mean value). The result, since it depends on the relation between the mass and luminosity of the star as a whole, will be substantially independent of assumptions regarding the superficial layers.

3. The photospheric pressure in a star depends essentially upon the opacity of the outer layers—as has been clearly shown by Milne.† The relations between pressure and density which hold good in these layers can be determined (in principle) with much greater accuracy than those in the deep interior. Whether they may be extended by extrapolation downward into the interior depends upon whether the composition changes with depth. Present evidence makes such a change appear probable. The problems of the interior and the exterior of a star seem to be almost independent, being connected only through the quantities M, L and r, which appear in both, and are fortunately "observable."

In conclusion, it is a pleasure to express my indebtedness to Professor Milne, whose courtesy in sending a copy of his latest paper before publication has led to the considerations which are here briefly stated.

Summary.—Replacement of the outer portions of a model star by material of arbitrarily different composition (so long as the mass involved is a negligibly small fraction of the whole) leads to a new model, likewise in equilibrium, and differing from the original very little in radius and luminosity or in density distribution, except near the surface. Consequences of this proposition are discussed.

Princeton University Observatory :
1937 *November* 3.

* Biermann, *A.N.*, **258**, 257, 1936.
† Milne, *M.N.*, **97**, 513, 1937.

ANNUAL CHANGE IN LONGITUDE: A CORRECTION.

Frank Schlesinger.

I am greatly indebted to Dr. G. Herck of the Leiden Observatory and to others for calling my attention to an error in my paper in *Monthly Notices*, **97**, 696, 1937. Formula (1) should read

$$X \sin \lambda \tan \phi + Y (\tan \phi_0 - \tan \phi \cos \lambda),$$

which is equivalent in this case to

$$+0{\cdot}79X + 1{\cdot}08Y.$$

The computed changes in longitude for various fractions of the year should be

0·0	$-$	^{s}·008	0·5	$+$	^{s}·011
0·1	$-$	6	0·6	$+$	7
0·2		0	0·7	$+$	3
0·3	$+$	6	0·8	$-$	5
0·4	$+$	11	0·9	$-$	10

These are somewhat smaller than those in the original paper, the range being ^{s}·021 instead of ^{s}·027. The conclusions are not altered: the polar motion accounts as satisfactorily as can be expected for the differences between Paris and Washington ; they also account, at least partially, for the differences between Greenwich and Washington.

Yale Observatory:
1938 *January* 10.

IONISATION IN STELLAR ATMOSPHERES.

Max Krook, Isaac Newton Student, Cambridge University.

1. The fundamental basis for the calculation of ionisation in stars is the Saha ionisation formula. This formula is deduced for a state of thermodynamic equilibrium, and the problem of its application to stellar atmospheres, where conditions deviate from thermodynamic equilibrium, has received some attention. Using a method developed by Milne in his researches on the statistical mechanics of the photo-electric effect, Woltjer * derived the general formula for the correction to be applied to the simple Saha formula, in terms of the relation between the local radiation field and the local temperature. From this Pannekoek † obtained a formula for the ionisation at the top of an atmosphere, and Woolley ‡ a formula for the ionisation as a function of optical depth. Russell's § tables of mean ionisations are computed from a Pannekoek formula and refer to the layer between $\tau = 0$ and $\tau = 0{\cdot}25$.

The deviations of the radiation field in a stellar atmosphere from a black-body distribution are due (*a*) to geometrical factors, and (*b*) to physical factors. These earlier derivations of ionisation formulæ include only the modification introduced by the geometrical factors. In the course of a programme of work now in progress at the Observatory, Cambridge, accurate expressions were required for the distribution of neutral and ionised atoms in a stellar atmosphere. On this account it was thought worth while to re-examine the problem on the wider basis to include also purely physical circumstances. Recent work by Greaves, Davidson and Martin || suggests strongly that geometrical considerations alone are not sufficient: it has been found that colour temperatures T_c of stars can, and do, differ very appreciably from effective temperatures T_e usually adopted, even in the visual region of the spectrum. It is the quality of the radiation in the ultra-violet which, along with the local temperature, determines the ionisation. If modification of the radiation field were due only to geometrical factors, this field would be describable simply by means of one ($\tau = 0$) or of two ($\tau > 0$) Planck functions and geometrical dilution factors. In particular we should then have $T_c \equiv T_e \equiv 2^{\frac{1}{4}} T_0$.

The identification in this way of the colour temperature over all ranges of frequency with the effective temperature is the basis of Pannekoek's formula for the ionisation at the top of a stellar atmosphere. He introduces it *via* a dilution factor $W = \frac{1}{2}$ to take into account the fact that radiation at the boundary of a star comes preferentially from directions within a solid angle 2π.

* *Nature*, March 14 (1925); April 11 (1925).
† Pannekoek, *B.A.N.*, **3**, 207, 1926.
‡ Woolley, *M.N.*, **93**, 691, 1933.
§ Russell, *Ap. J.*, **75**, 337, 1932. || *M.N.*, **94**, 488, 1934.

Woolley in his formula for the ionisation at all depths used the linear flow approximation to obtain the intensities of the two streams, and assigned to the outward stream a temperature given by its amount, and to the inward stream the local temperature. The radiation field at a point is in this way represented by a linear combination of two Planck functions with arguments and coefficients determined geometrically.

Russell replaces the dilution factor $\frac{1}{2}$ of Pannekoek's formula by a mean value in the region $\tau = 0$ to $\tau = 0{\cdot}25$, and, as mentioned above, his tables refer to this range of optical depth.

In this paper we shall adopt the probably more fundamental standpoint that the radiation field is completely determined by the dependence of the coefficient of continuous absorption k_ν on frequency ν. As a matter of fact, the solution, with the appropriate function k_ν and the appropriate boundary conditions, of the equation of transfer of radiation contains explicit reference to both the physical and the geometrical factors. In brief, the procedure then is to solve the equations of transfer for an arbitrary dependence of k_ν on ν and then to evaluate the integrals of the Woltjer formula for this radiation field.

2. At this stage it might be as well to state the assumptions we shall make. These are that :

(i) Transfer in local thermodynamic equilibrium holds for the frequencies of the continuous spectrum.

(ii) The continuous absorption coefficient k_ν can be written in the form

$$k_\nu = n_\nu \bar{k},$$

where n_ν is independent of depth but may depend on ν (i.e. k_ν is the same function of frequency at all depths); $\bar{k}$ is the mean continuous absorption coefficient which determines the temperature distribution

$$T^4 = T_0^4(1 + \tfrac{3}{2}\tau), \tag{1}$$

where the general optical depth τ is defined by

$$\tau = \int_x^0 \bar{k}\rho dx.$$

It may be noted here that this average $\bar{k}$ is not the Rosseland mean $\bar{k}_1$ as usually defined for the stellar interior :

$$\frac{1}{\bar{k}_1} = \int_0^\infty \frac{1}{k_\nu} \cdot \frac{dB_\nu}{dT} d\nu \div \int_0^\infty \frac{dB_\nu}{dT} d\nu,$$

where B_ν is the Planck function.

From the equation of transfer in its most general form and the equation of radiative equilibrium, a definition of the mean absorption coefficient consistent with the relation $\mathcal{J}(\tau) = H(2 + 3\tau)$ is

$$\bar{k} = \frac{1}{H}\int_0^\infty k_\nu H_\nu \, . \, d\nu, \tag{1a}$$

i.e. $\bar{k}$ is the arithmetic mean of the k_ν with the associated fluxes H_ν as weight factors. Equivalent definitions are

$$\frac{1}{\bar{k}}\frac{dK}{dT}=\int_0^\infty \frac{1}{k_\nu}\cdot\frac{dK_\nu}{dT}d\nu$$

and, with the approximation $K_\nu=\frac{1}{3}\mathcal{J}_\nu$,

$$\frac{1}{\bar{k}}\cdot\frac{d\mathcal{J}}{dT}=\int_0^\infty \frac{1}{k_\nu}\cdot\frac{d\mathcal{J}_\nu}{dT}d\nu. \tag{1b}$$

From (1*b*) we see that $\bar{k}$ coincides with $\bar{k}_1$ if the local radiation field is identified with a black-body distribution at the local temperature. This is not the case in a stellar atmosphere, and so the definition (1*a*) is the appropriate one.

In addition to these assumptions, the Eddington approximation $K_\nu=\frac{1}{3}\mathcal{J}_\nu$ and the approximate boundary condition $\mathcal{J}_\nu(0)=2H_\nu(0)$ are applied. The notation is the same as in Strömgren's article in the *Handbuch der Astrophysik*.

The equation of transfer for radiation of frequency ν in the continuous spectrum for material stratified in plane layers and with k_ν of the assumed form is

$$\cos\theta\cdot\frac{dI_\nu}{d\tau}=n_\nu\{I_\nu-B_\nu(\tau)\},$$

where $B_\nu(\tau)$ is the Planck function corresponding to the temperature given by (1). By the usual methods, with $K_\nu=\frac{1}{3}\mathcal{J}_\nu$, this leads to the second-order differential equation

$$\frac{d^2\mathcal{J}_\nu}{d\tau^2}=3n_\nu^2\{\mathcal{J}_\nu-B_\nu(\tau)\}. \tag{2}$$

The solution of (2) with the conditions that $\mathcal{J}_\nu(\tau)$ must not tend exponentially to infinity as $\tau\to\infty$ and that $\mathcal{J}_\nu(0)=2H_\nu(0)$ is then

$$\left.\begin{aligned}\mathcal{J}_\nu(\tau)=&\frac{1-\frac{\sqrt{3}}{2}}{1+\frac{2\sqrt{3}}{3}}\int_0^\infty B_\nu(t)e^{-\sqrt{3}n_\nu(\tau+t)}n_\nu dt\\&+\frac{\sqrt{3}}{2}\int_\tau^\infty B_\nu(t)e^{-\sqrt{3}n_\nu(t-\tau)}n_\nu\,.\,dt\\&+\frac{\sqrt{3}}{2}\int_0^\tau B_\nu(t)e^{\sqrt{3}n_\nu(t-\tau)}n_\nu\,.\,dt.\end{aligned}\right\} \tag{3}$$

This determines the radiation field as a function of optical depth, the associated local temperature being specified by (1). After a little rearrangement of terms, (3) reduces to

$$\left.\begin{aligned}\mathcal{J}_\nu(\tau)=A(\tau)&\int_0^\infty B_\nu(t)e^{-\sqrt{3}n_\nu t}dt\\&-\sqrt{3}n_\nu\int_0^\tau B_\nu(t)\sinh\{\sqrt{3}n_\nu(\tau-t)\}dt,\end{aligned}\right\} \tag{4}$$

with

$$A(\tau)=n_\nu\left\{\frac{1-\frac{\sqrt{3}}{2}}{1+\frac{2\sqrt{3}}{3}}e^{-\sqrt{3}n_\nu\tau}+\frac{\sqrt{3}}{2}e^{\sqrt{3}n_\nu\tau}\right\}. \tag{5}$$

3. The ionisation of a gaseous element of ionisation potential I in thermodynamic equilibrium at temperature T and electron pressure P is given by the Saha formula

$$\log_{10}\left(\frac{xP}{1-x}\right)=-\frac{5040I}{T}+\tfrac{5}{2}\log T-6{\cdot}49+\log\left\{\frac{2b'(T)}{b(T)}\right\}, \tag{6}$$

where

x = the fraction of atoms ionised,
$b(T)$ = the partition function for the neutral atom, and
$b'(T)$ = the partition function for the ionised atom.

For the case of a departure from thermodynamic equilibrium Woltjer * gives the correction to be applied in the form

$$\frac{xP}{1-x}=\left(\frac{xP}{1-x}\right)_T\frac{\int_{\nu_0}^{\infty}\psi(\nu)\mathcal{J}_\nu d\nu}{\int_{\nu_0}^{\infty}\psi(\nu)e^{-h\nu/kT}\left(\frac{2h\nu^3}{c^2}+\mathcal{J}_\nu\right)d\nu}, \tag{7}$$

where the suffix T indicates that the relevant quantity is to be evaluated for the state of thermodynamic equilibrium at the local temperature, T, and

$\mathcal{J}_\nu$ = (as before) the mean intensity of ν-radiation,
$\psi(\nu)$ = the probability coefficient for ionisation of the atom by a quantum of frequency ν,
ν_0 = the frequency of the series limit, i.e. $h\nu_0=I$.

For all cases of practical interest $2h\nu^3/c^2 >> \mathcal{J}_\nu$, and so the term involving $\mathcal{J}_\nu$ in the denominator can be neglected. The correction factor $f(\tau)$ can then be written

$$f(\tau)=\frac{\int_{\nu_0}^{\infty}\nu^{-3}\mathcal{J}_\nu(\tau)d\nu}{\int_{\nu_0}^{\infty}\nu^{-3}e^{-h\nu/kT}\cdot\frac{2h\nu^3}{c^2}d\nu},$$

i.e.

$$f(\tau)=\frac{c^2}{2kT}e^{\frac{h\nu_0}{kT}}\int_{\nu_0}^{\infty}\nu^{-3}\mathcal{J}_\nu(\tau)d\nu, \tag{8}$$

where we have as usual assumed $\psi(\nu)$ to be proportional to ν^{-3}.

The evaluation of $f(\tau)$ with the $\mathcal{J}_\nu(\tau)$ of (4), corresponding to an arbitrary form of variation of n_ν with ν, may be a matter of considerable difficulty.

* *Physica*, 5, 406, 1925.

The subsequent discussion will thus be confined to two cases of physical interest.

4. *The Case $k_\nu = constant$.*—In the first instance we trace out the consequences of a continuous absorption coefficient k_ν independent of ν, i.e. $n_\nu \equiv 1$ in (4). The correction factor (8) then becomes

$$f(\tau) = \frac{c^2 e^{\frac{h\nu_0}{kT}}}{2kT} \int_{\nu_0}^{\infty} \nu^{-3} d\nu \left\{ A(\tau) \int_0^{\infty} B_\nu e^{-\sqrt{3}t} dt - \sqrt{3} \int_0^{\tau} B_\nu \sinh [\sqrt{3}(\tau - t)] dt \right\}, \quad (9)$$

where

$$A(\tau) = \left\{ \frac{1 - \frac{\sqrt{3}}{2}}{1 + \frac{2\sqrt{3}}{3}} e^{-\sqrt{3}\tau} + \frac{\sqrt{3}}{2} e^{\sqrt{3}\tau} \right\} \quad (10)$$

and the Planck function

$$B_\nu(t) = \frac{2h\nu^3}{c^2} \left(e^{\frac{h\nu}{kT}} - 1 \right)^{-1}.$$

The order of integration with respect to ν and t may now be inverted in (9) and the integration with respect to ν carried out. This leads to

$$f(\tau) = -\frac{e^{\frac{h\nu_0}{kT}}}{T} \left\{ A(\tau) \int_0^{\infty} T \log \left(1 - e^{-\frac{h\nu_0}{kT}} \right) e^{-\sqrt{3}t} dt \right.$$
$$\left. - \sqrt{3} \int_0^{\tau} T \log \left(1 - e^{-\frac{h\nu_0}{kT}} \right) \sinh [\sqrt{3}(\tau - t)] dt \right\}, \quad (11)$$

where the temperature T within the integral signs is the local temperature at depth t. For most practical applications and in the temperature range responsible for the major contributions to these integrals, $\frac{h\nu_0}{kT} >> 1$, so that we can write as a sufficiently close approximation :

$$-\log \left(1 - e^{-\frac{h\nu_0}{kT}} \right) = e^{-\frac{h\nu_0}{kT}}.$$

(11) then becomes

$$f(\tau) = \frac{e^{\frac{h\nu_0}{kT}}}{T} \left\{ A(\tau) \int_0^{\infty} T e^{-\left(\sqrt{3}t + \frac{h\nu_0}{kT} \right)} dt - \sqrt{3} \int_0^{\tau} T e^{-\frac{h\nu_0}{kT}} \sinh \{\sqrt{3}(\tau - t)\} dt \right\}, \quad (12)$$

These integrals cannot be evaluated exactly, and so recourse must be had to the use of an asymptotic expansion or to numerical integration. We shall not reproduce the details of the calculation here, but quote a number of results for a typical case. Correction factors at $\tau = 0$ and $\tau = 0{\cdot}5$ for the case of boundary temperature $T_0 = 5040°$ (and so effective temperature $T_e = 5972°$) and for three elements, *Na* ($I = 5{\cdot}09$ volts), *Fe* ($I = 7{\cdot}63$ volts), *H* ($I = 13{\cdot}54$ volts), are exhibited in the accompanying Table I. For the

purposes of comparison, values computed from Woolley's formula (which reduces to Pannekoek's result at $\tau = 0$) are included in the table.

TABLE I

Element	$f(0)$ by (12)	$f(0)$, Woolley	$f(\cdot 5)$ by (12)	$f(\cdot 5)$, Woolley
Na	4·86	5·43	1·90	2·25
Fe	20·2	16·4	3·86	4·12
H	1100	216	36·3	17·7

It will be observed that the factor given by (12) is different from that deduced from the Pannekoek or Woolley formulæ, the discrepancy increasing the larger the ionisation potential of the element. This is accounted for by the fact that for $k_\nu = \text{constant}$ the colour temperature exceeds the effective temperature even in the visual region, and this excess increases towards the ultra-violet frequencies, which are the ones effective in producing the ionisation. It is easy to see physically why this is the case. The intensity of the ultra-violet radiation is small and at the same time very sensitive to small changes in temperature. Thus contributions from deeper layers of the atmosphere, even though damped by absorption in the same way, come in with greater weight than they do in the visual region. This means that the average depth at which the far ultra-violet radiation originates is greater and so corresponds to a higher temperature.

The correction factor decreases rapidly from the surface inwards, corresponding to a closer approach to thermodynamic equilibrium conditions in the sense that the local mean radiation intensity $\mathfrak{J}_\nu(\tau)$ tends towards a Planck distribution at the local temperature. Also, at the surface, the geometrical factors exert a greater effect than in the interior and in just the direction of increasing the correction.

The results cited above show that the geometrical considerations of the Pannekoek and Woolley formulæ must be supplemented by physical considerations. We next discuss a case where the increase of the continuous absorption coefficient k_ν towards the violet is allowed for in a rough sort of manner.

5. *k_ν represented by a Step Function.*—The continuous absorption coefficient in a stellar atmosphere can be regarded as made up of two parts:

(1) The metallic absorption, which increases towards the ultra-violet due to the increase in the number of series heads, and

(2) The hydrogen absorption, which varies discontinuously, falling off as the inverse cube of the frequency between the series heads and experiencing a large increase as a new limit is introduced.

It was inferred by Milne from Abbot's observations of darkening to the limb that k_ν is independent of ν for the Sun. This leads also to a

theoretical colour temperature agreeing closely with the observed value.* Pannekoek,† on the other hand, calculating k_ν on the basis of Kramers' formula, obtained at λ 4500 a colour temperature of about 4300°, which is much too low. This discordance is probably due to the breakdown of the assumption that Kramers' theory, which is in fact derived only for hydrogen-like atoms, applies also to the optical spectra of metallic elements. Woolley ‡ has pointed out that a consideration of the oscillator-strengths f for atomic transitions, combined with the sum rules, leads to a determination of k_ν. Unfortunately, these f-values are as yet known in only a few cases, and so the method, which is independent of Kramers' theory, is not generally applicable. In the case of the principal series of sodium, § for which oscillator strengths have been calculated and also determined experimentally, the results are very different from those of Kramers' theory. The procedure adopted here for estimating k_ν uses as starting-point Milne's observation of the constancy of k_ν in the visible region for the Sun. It is essentially different from that of Biermann, Pannekoek, etc., who base their considerations on Kramers' theory.

In stars of type near Go, k_ν is approximately constant in the visual region of the spectrum. This effect may be regarded as due to the balancing-out of the increase in metallic absorption against the decreasing hydrogen absorption beyond the head of the Paschen series. § As soon as the Balmer limit is reached, the absorbing power of hydrogen increases enormously by virtue of the facts that the Paschen absorption has at this stage almost completely disappeared, and that atoms in 2-quantum states, which are much more highly populated than the 3-quantum states, are now in a position to contribute. This results in a discontinuous increase in k_ν at the Balmer limit. After this point the hydrogen absorption again falls off rapidly. It is probably not a very bad approximation to assume that, between the Balmer and Lyman limits, the increasing metallic absorption on the average again balances out the decreasing hydrogen contribution, so that k_ν can be regarded as constant in this range as well. At the Lyman limit the full absorbing power of hydrogen (itself estimated to be very much more abundant than the other elements) comes into play. By far the largest fraction of hydrogen atoms are in the ground state, and these now become available as absorbing mechanisms. From this point towards the far ultra-violet we again suppose the superposition of metallic and hydrogen contributions to produce approximate constancy of k_ν.

We have thus obtained a representation of the continuous absorption coefficient as a step function, constant between the series limits of hydrogen and increasing discontinuously at each such limit. An estimate of the relationship between these constant values will be made later. We shall further suppose that in the averaging process for k_ν the main weight is thrown into the visual region, so that the k_ν of this range is appreciably the

* Woolley, *M.N.*, **95**, 101, 1934 ; and Chandrasekhar, *ibid.*, **96**, 21, 1935.
† Pannekoek, *M.N.*, **95**, 529, 1935.
‡ Woolley, *The Observatory*, **58**, 297, 1935.
§ *Cf.* for example Woolley, *M.N.*, **94**, 713, 1934.

same as the averaged $\bar{k}$, *i.e.* this k_ν determines the temperature distribution. Our description of the radiation field $\mathcal{J}_\nu(\tau)$ now involves a coefficient n_ν which is

(1) Constant and approximately equal to 1 up to the Balmer limit,
(2) Constant and $=n_1$ between the Balmer and Lyman limits,
(3) Constant and $=n_2$ beyond the head of the Lyman series,

i.e.

$$\mathcal{J}_\nu(\tau) = A_i(\tau)\int_0^\infty B_\nu(t)e^{-\sqrt{3}n_i t}dt - \sqrt{3}n_i\int_0^\tau B_\nu(t)\sinh\{\sqrt{3}n_i(\tau-t)\}dt \qquad (13)$$

with

$$A_i(\tau) = \left\{\frac{1-\frac{\sqrt{3}}{2}}{1+\frac{2\sqrt{3}}{3}}e^{-\sqrt{3}n_i\tau} + \frac{\sqrt{3}}{2}e^{\sqrt{3}n_i\tau}\right\}n_i, \qquad (14)$$

where the suffix

$i=1$ for $\nu_1 =$ Balmer limit $< \nu <$ Lyman limit $=\nu_2$,
$i=2$,, $\nu >$ Lyman limit $=\nu_2$.

The first ionisation potential of most elements is such that $\nu_1 < \nu_0 \leqslant \nu_2$, so that for these the integral of (8) separates into two parts:

$$\int_{\nu_0}^\infty \nu^{-3}\mathcal{J}_\nu(\tau)d\nu = \int_{\nu_0}^{\nu_2}\nu^{-3}\mathcal{J}_\nu(\tau)d\nu + \int_{\nu_2}^\infty \nu^{-3}\mathcal{J}_\nu(\tau)d\nu, \qquad (15)$$

where in the first integral $i=1$ and in the second $i=2$. By steps exactly analogous to those used in deducing (12) we obtain from (13) and (15):

$$\left.\begin{aligned} f(\tau) = \frac{e^{\frac{h\nu_0}{kT}}}{T}\Bigg\{ & A_1(\tau)\int_0^\infty T\left(e^{-\frac{h\nu_0}{kT}} - e^{-\frac{h\nu_2}{kT}}\right)e^{-\sqrt{3}n_1 t}dt \\ & - \sqrt{3}n_1\int_0^\tau T\left(e^{-\frac{h\nu_0}{kT}} - e^{-\frac{h\nu_2}{kT}}\right)\sinh\{\sqrt{3}n_1(\tau-t)\}dt \\ & + A_2(\tau)\int_0^\infty Te^{-\frac{h\nu_2}{kT}}e^{-\sqrt{3}n_2 t}dt \\ & - \sqrt{3}n_2\int_0^\infty Te^{-\frac{h\nu_2}{kT}}\sinh\{\sqrt{3}n_2(\tau-t)\}dt\Bigg\}. \end{aligned}\right\} \qquad (16)$$

Again, these integrals can be evaluated only by means of an asymptotic expansion or by numerical integration. For elements of ionisation potential less than, and not too close to, 13·54 volts it can easily be shown that the error introduced by dropping the last two integrals in (16) and by neglecting the terms in $e^{-\frac{h\nu_2}{kT}}$ in the first two integrals is negligible. For this case we can then write

$$f(\tau) = \frac{e^{\frac{h\nu_0}{kT}}}{T}\left\{A_1(\tau)\int_0^\infty Te^{-\frac{h\nu_0}{kT}}e^{-\sqrt{3}n_1 t}dt - \sqrt{3}n_1\int_0^\tau Te^{-\frac{h\nu_0}{kT}}\sinh\{\sqrt{3}n_1(\tau-t)\}dt\right\}. \qquad (17)$$

For elements of ionisation potential greater than or equal to 13·54 volts only the last two integrals of (16) are relevant, so that $f(\tau)$ is given by a formula exactly equivalent to (17) with $A_2(\tau)$ replacing $A_1(\tau)$ and n_2 replacing n_1.

It is possible to prove formally that when n is large in (17), $f(\tau)$ tends to unity very rapidly as τ increases from zero, and that in the limiting case when $n \to \infty$, $f(\tau) = 1$ for all $\tau > 0$. At the boundary $\tau = 0$ the correction factor may still be different from unity (this is due to the preponderating effect of the geometrical factor here). An approximate expression for n large and one suitable for general discussion is

$$f(\tau) = 1 - \tfrac{1}{2}e^{-\sqrt{3}n\tau}\left[1 - \left(1 + \frac{h\nu_0}{kT}\right)\left(\frac{3}{8}\tau - \frac{\sqrt{3}}{8n}\right)\right]. \tag{18}$$

Even for moderate values of n the range of optical depth for which the second term of (18) remains comparable with unity is very small.

We can easily convince ourselves of this by evaluating the factor $f(\tau)$ for a number of cases by (18), where the exponential factor $e^{-\sqrt{3}n\tau}$ is responsible for the rapid fading out of the second term for large n. As a rough guide, for those values of τ for which $n\tau \geqslant 1$, the factor $f(\tau)$ is sufficiently close to 1 to permit of the application of the simple Saha formula without correction.

For a star of effective temperature about 6000°, the balancing-out of metallic and of hydrogen Paschen absorption in the visible region of the spectrum leads, with the calculated oscillator strengths for hydrogen, to estimates n_1, n_2 of orders 10 and 10^8 respectively. It then follows that for all ionisation potentials of physical interest the Saha formula requires correction only in a layer of very small optical depth at the top of the atmosphere (where the unidirectional effect predominates). In this shallow boundary layer the factor is less than 1, and approaches 1 very rapidly with increasing optical depth. To illustrate this we reproduce values of $f(\tau)$ at $\tau = 0$ and $\tau = 0{\cdot}1$ calculated from (17) with $n_1 = 10$:

For

$$Na\ (I = 5{\cdot}09 \text{ volts}) :\ f(0) = {\cdot}71\ ;\ f(0{\cdot}1) = 1{\cdot}01,$$

and for

$$Fe\ (I = 7{\cdot}63 \text{ volts}) :\ f(0) = {\cdot}83\ ;\ f(0{\cdot}1) = 1{\cdot}04.$$

The factor at $\tau = 0{\cdot}1$ is in both cases 1 within the limits of error of computation. Further, the ionisation in the topmost layer is less than in thermodynamic equilibrium for these cases. For hydrogen the recovery to the ordinary Saha formula is still more striking.

Even with less charitable estimates of n_1 and n_2 the same general features as mentioned above are reproduced. It is also reasonable to suppose that these conclusions are valid when k_ν is represented, not by a step function in this particular way, but by any function of ν, provided it increases sufficiently rapidly towards the violet, as we have reason to believe it to do. The quick return of the ionisation to normal as in thermodynamic equilibrium is a consequence of k_ν increasing with ν.

6. *Conclusions.*—For the case k_ν constant with ν the Saha formula requires a correction which may become considerable (*cf.* Table I). If, however, due allowance is made for an increase of the continuous absorption coefficient towards the ultra-violet, corrections, if any, to the simple Saha formula are confined only to a layer of small optical depth (measured in terms of the general depth τ) at the top of the atmosphere. For most astrophysical purposes the effect of the correction in this boundary layer is of no practical importance. In the formation of absorption lines, for example, this layer will have an influence only on the central intensity, but will not affect the wings of the line or, for strong lines, the equivalent width. In addition, the correction at the top of the atmosphere in the case treated here is in the opposite direction to that deduced by Pannekoek and Woolley, and so to drop the correcting factor would be to steer a middle course between the two. It thus appears that the simple Saha formula without modification may safely be applied to calculating the ionisation in a stellar atmosphere—a gratifying simplification.

It is interesting to inquire whether the restoration of Saha's formula, despite the non-equilibrium character of the stellar atmosphere, occurs inevitably or whether it is a contingency dependent on the particular maximum of the solar energy curve among the particular series limits of a very abundant material, hydrogen. Our equations show that it is not altogether a contingency. For example, in a star of higher surface temperature than type G, say type A, having an effective temperature of about 12,000°, the greatest weight of the averaged k_ν is thrown into the region $\nu_1 < \nu < \nu_2$, so that the factor n drops from 10 to 1, in which case the ionisation of the *metals* apparently requires corrections something like those appropriate to k_ν constant. But the metals have been ionised before we get to this type, and, in general, an element can only escape ionisation if its ionisation potential lies considerably beyond the wave-length of the maximum of the stellar atmosphere. Accordingly, if the correction is large, the case cannot be one of importance, as there cannot be any strong lines in the class requiring correction.

Summary.—Corrections to the Saha ionisation formula consequent upon the departure from thermodynamic equilibrium in stellar atmospheres are investigated. It is found that if the continuous absorption coefficient is constant throughout the spectrum, corrections can be very considerable, but it is believed that a case of much more physical interest is that in which the coefficient increases strongly with frequency in the remote ultra-violet, and in this case the corrections are negligible.

In conclusion, it is a pleasure to record my thanks to Professor Sir A. S. Eddington and to Dr. R. v. d. R. Woolley for their kind advice in the course of this work. Dr. Woolley also kindly assisted in reading the manuscript and in pointing out several important considerations.

Caius College, Cambridge :
1938 *January* 6.

NOTE ON THE STATE OF THE OUTER PLANETS.

Harold Jeffreys, D.Sc., F.R.S.

Mr. Peek's paper * suggests to me that it is time that two possible misinterpretations of my papers on the outer planets † were corrected.

Measures of the dark radiation of Jupiter and Saturn were published first in the interval between these two papers, and the second dealt partly with the fact that the observed radiation was appreciably larger than could be explained by the absorption and re-emission of solar radiation. I showed that to explain it by radioactivity would require about 1300 times as much uranium per unit surface as is present on the Earth. It seems to have been supposed that I was seriously advocating this as the correct explanation, whereas I was merely showing how much radioactivity would be needed if this was to be accepted as the explanation. I had no satisfactory explanation at the time and have none now, but I thought that the improbability of such an amount of radioactive material near the surface was sufficiently evident without elaboration. I pointed this out in a later note,‡ which seems to have been overlooked. The other alternative mentioned in this note was departure from black-body radiation, but as what was measured was the whole radiation, the temperatures reported being only an interpretation of this, it seems unlikely that this is the correct one. I discussed the possibility of various systematic errors in the observations with Menzel several years ago, but they all appeared to have been foreseen.

A further alternative appears to be worth consideration. My original argument showed that with a surface temperature of 300° absolute, which was lower than anybody had considered up to that time, Jupiter would have had time to lose about $1{\cdot}5 \times 10^5$ calories per gram of its mass. But if we take the actual temperature of about 150° absolute and allow for solar radiation, this must be divided by about 30, and 5000 calories per gram does not appear a prohibitive amount. Internal temperatures of the order of some thousands of degrees are to be expected even in the Earth, as a relic of original heat. With suitable internal conditions it seems that original heat may explain the slight difference between the calculated and observed radiations.

My suggestion of a thick ice layer in these planets appears to have been responsible for the belief, mentioned in Mr. Peek's paper, that the interiors are cold. I do not think, however, that this can be maintained. If we assume a heated original state at all, upward transfer of heat implies a downward increase of temperature, whether transfer is by conduction, convection, or radiation. In the early stages at least it would be by convection, which implies a minimum temperature gradient of some degrees per kilometre and a central temperature of the order of 10^5 degrees. If the gradient fell below

* *M.N.*, **97**, 574–582, 1937.
† *M.N.*, **83**, 350–354, 1923 ; **84**, 534–538, 1924.
‡ *B.A.A. Journal*, **36**, 153, 1926.

this, convection would cease and further cooling would be negligible. Thus a central temperature of this order of magnitude appears to be needed in any case. My actual reason for suggesting ice was based on analogy with the satellites, which suggested at any rate that there should be a large quantity of matter with a density near 1. I had not considered the internal temperatures at all when the papers were written, and "ice" was not intended as a positive identification but merely as a convenient part of a model designed to test whether, with any reasonable distribution of density, the data about the mean densities and the ellipticities could be satisfied. Wildt, whose model takes account of much more information, suggested later that the ice would be in the form of Bridgman's high-pressure modification, Ice VI, which can exist at fairly high temperatures ; I understand from a letter that Bridgman has found another form, Ice VII, at still higher pressures and temperatures, but these still do not reach what may be expected at the relevant depths in the planets.

Present theory and observation both concern only the surface temperatures, or rather those of the effective radiating layers (which on the Earth would be at a height of some kilometres). Extension to the interiors would require a quantitative examination of the adiabatic temperature gradients and probable rates of cooling in highly compressed gases, and possibly a theoretical examination of the crystal state of ice at pressures of the order of 2×10^{12} c.g.s. If internal cooling really contributes appreciably to the surface temperature it may have to be at levels where convection can take place, and there is no way of ascertaining at present how much cooling can be going on at such levels. I think that it is desirable that this alternative should be tested, but more of the necessary knowledge is available to others than to me.

OBSERVED PHOTOGRAPHIC MAGNITUDES OF THE SUPER-NOVA 145·1937 PERSEI IN N.G.C. 1003.

A. Beer, Ph.D., and C. C. L. Gregory, M.A., D.I.C.

The position of this super-nova is R.A. $2^h\ 35^m\ 21^s{\cdot}2$, Dec. $+40°\ 36'\ 2''$ (1937·0). It is situated in the east end of N.G.C. 1003.

Photographs of the nova were obtained on eight nights with the 24-inch Wilson reflector of the University of London Observatory at Mill Hill. The photographs were taken in the principal focus of the telescope on Ilford Iso-Zenith plates, backed (speed 700 H. & D.). The plates were developed with metol-hydrochinone for 3 minutes.

In the case of the plate taken on October 4, exposures were made on the stars of the North Polar Sequence both before and after exposing on the nova. The duration of the exposures was the same—viz. 20 minutes—and the exposure on the nova was made when it was at the same altitude as the pole field.

In order to determine the photographic magnitudes a curve was obtained relating the square roots of the diameters of twenty stars of the North Polar Sequence to their magnitudes as given in *Trans. I.A.U.*, **1**, 71. The stars used were : Nos. 13, 15–27, 11*r*, 12*r*, 5*s*, 7*s*, 8*s* and 9*s*, both exposures being measured. From the curve thus obtained the magnitudes of a number of stars in the immediate field of the nova were determined from their measured diameters. These comparison stars then served to determine the magnitude of the nova on the remaining plates. The diameters of the star images were measured in every case twice in each of two perpendicular directions.

The following table of the comparison stars gives their positions, relative to the super-nova, and their magnitudes. The values in brackets are the improved magnitudes obtained by very accurate photographic determinations made at the Hamburg-Bergedorf Observatory by Dr. A. A. Wachmann, to whom we are much obliged for his kind information. The magnitudes of the bright stars *a*, *b*, *c*, *d* are included for the sake of completeness only, and have been taken from the *Helsingfors Astrographic Catalogue*, vol. i, fasc. 3, p. 203, Nos. 340, 360, 368, 369.

Table I

Star	X	Y	m	Star	X	Y	m
	′	′	m		′	′	m
a	+ 3·1	−1·0	9·2	*n*	+5·3	+3·8	14·2
b	− 5·1	−4·2	9·7	*o*	−5·3	+4·8	(14·5)
c	−10·3	+0·1	10·3	*p*	+6·3	−3·2	14·6
d	−10·2	−6·9	10·4	*q*	−4·3	−0·2	14·6
e	− 1·6	+8·5	(12·1)	*r*	−5·7	+4·4	(14·7)
f	− 1·6	+6·7	(12·7)	*s*	−3·4	+5·6	14·9
g	− 2·1	+2·7	(13·2)	*t*	+3·0	−2·7	15·0
h	+ 0·2	+0·5	(13·3)	*u*	−3·5	+5·1	15·1
i	− 2·4	+6·5	(13·4)	*v*	−2·4	+3·4	15·2
j	+ 2·0	−2·8	(13·5)	*w*	+3·5	+1·3	15·2
k	− 2·3	+8·1	(13·5)	*x*	+1·2	−1·6	15·2
l	+ 4·0	+3·8	13·8	*y*	−0·7	+2·3	15·3
m	+ 6·3	−2·6	14·0	*z*	−0·2	+2·3	15·8

The observed magnitudes of the super-nova, together with particulars regarding the plates, are given in the following table :—

Table II

	U.T.	Exp.	Mag.	Obs.
	h m	m		
1937 Sept. 20	23 56·8	13·5	13·3	G
21	0 25·2	15	13·5	G
26	21 56·0	30	13·9	G
Oct. 3	22 36·2	24·5	14·6	G
4	23 7·9	20	14·8	G
9	20 39·2	70	15·4	B
10	0 49·4	25·3	15·2	B
15	22 0·6	70	(15·8)	B

An observation kindly communicated by Mr. M. Beyer-Hamburg, who observed the visual magnitude to be 13·9 on October 12 when compared with the light curve derived from the above results, gives the rather high colour index of +1·7.

University of London Observatory :
1937 *November* 3.

THE INCLINATIONS TO THE LINE OF SIGHT OF THE NON-GALACTIC NEBULÆ.

F. G. Brown.

It has been shown by J. H. Reynolds * that if the spiral nebulæ are regarded as approximating to thin discs, there is an excess of small inclinations of the large spirals to the line of sight. Later, Mr. Reynolds,† using material collected by the writer, concluded that this excess of small inclinations also exists in the case of nebulæ of moderate size, down to 3′ diameter, although in a smaller degree.

Soon after the publication of the latter paper, E. Öpik ‡ suggested that the observed excess is merely the result of photographic selection. Since then, however, important additions have been made to our knowledge of the form, dimensions and brightness of the non-galactic nebulæ, notably the Harvard and Heidelberg lists. Thus, not only is more complete material provided for the discussion of the inclinations, but it is also possible to test the effect of such forms of observational selection as might conceivably occur.

The present study is based, primarily, on the catalogue of Herschel nebulæ north of declination $-20°$, measured at Heidelberg on plates taken with the Bruce 40-cm. portrait lens and the 70-cm. reflector. § From this catalogue all non-galactic nebulæ of 2′ diameter and greater have been extracted and classified—first into columns according to size, and then into three groups with axis ratios b/a lying between the limits 0 and 1/3, 1/3 and 2/3, 2/3 and unity respectively. The few objects with axis ratios exactly 1/3 have been apportioned equally between the first and second groups, and those with axis ratios exactly 2/3 between the second and third groups. A more detailed sub-division is of doubtful value, since, on the one hand, the outline of a nebula seen in plan is not truly circular, and on the other hand, the thinnest spiral has an appreciable polar thickness.

The results of the analysis are given in Table I, which reveals a large excess of small inclinations to the line of sight, although, if the planes were distributed at random, there should be equal numbers of nebulæ in each line. Actually, objects with small inclinations appear to be twice as numerous as those seen nearly in plan, in spite of the fact that the analysis includes certain elliptical or spheroidal nebulæ with polar thickness exceeding one-third of the equatorial diameter. These are automatically added to the second and third lines of the table, even when the equatorial plane is nearly in the line of sight.

* *M.N.*, **71**, 129, 1920.
† *M.N.*, **82**, 510, 1922.
‡ *Observatory*, **46**, February and May 1923.
§ *Publications of the Heidelberg Observatory* (*Königstuhl*), **9**, 1926.

Table I

Analysis of the Nebulæ in the "Heidelberg Catalogue of Herschel Nebulæ," with Axis Ratios between Certain Limits

Axis Ratio b/a	Limits of Major Axis				Total
	8′ and greater	4′ to < 8′	3′ to < 4′	2′ to < 3′	
0 to 1/3	17	60	70	129	276
1/3 to 2/3	13	43	44	103	203
2/3 to 1	6	22	25	61	114
Total	36	125	139	293	593
Mean value of b/a	·390	·421	·425	·444	

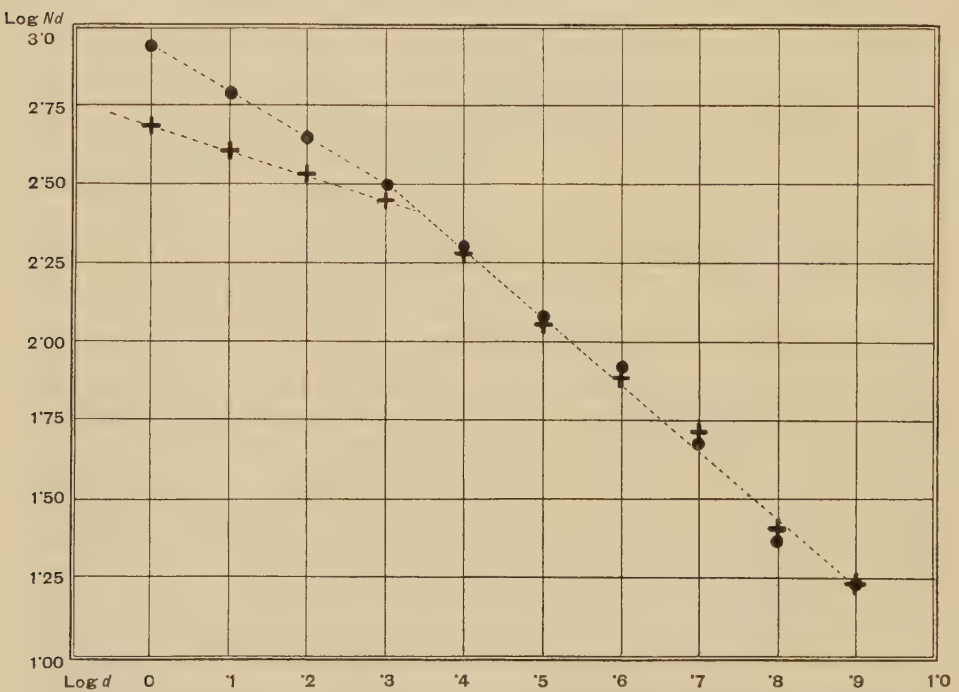
The remaining part of this paper is devoted to an investigation of the problem, which immediately arises, whether the excess is real or whether it results from some form of observational selection.

Homogeneity and Completeness of the Catalogue.—The *Heidelberg Catalogue* is based on the general catalogue of Sir John Herschel, and to the limiting diameter of 2′ may be regarded as reasonably complete and representative. Within the area covered by the catalogue there are known to the writer 109 other objects to this limiting diameter, and it is considered unlikely that more than a further 10 per cent. of such objects remain to be discovered. Of these 109 objects, 49 or 45 per cent. have axis ratios lying between 0 and 1/3, the percentage differing only 1 per cent. from that of the catalogue objects ; so that a final distribution of the small, undiscovered residue among the groups should have little effect on the above proportion of large to small inclinations.

The degree of completeness of the Heidelberg list may also be inferred from the diagram, in which logarithms of the cumulative totals of nebulæ Nd, to successive limiting diameters d are plotted against log d, for values of b/a between 1/3 and unity (black dots) and for values between 0 and 1/3 (crosses). In the former group the relationship is nearly linear down to 2′ diameter, after which there is a progressive falling off in the expected numbers ; in the latter group the falling off begins slightly earlier and, below 2′ diameter, proceeds much more rapidly. This more rapid falling off is doubtless due to the fact that the integrated brightness of the average nebula, when the axis ratio lies between 0 and 1/3, is more than a magnitude fainter than that of a similar object seen nearly in plan. In the same way an elliptical nebula is considerably brighter than a spiral of the same general form and dimensions. It is for this reason that a limiting diameter has been adopted in preference

to a limiting magnitude ; for in the latter case a disproportionate number of small bright elliptical nebulæ and round spirals are admitted to the lists.

Correlation between Inclination and Measured Diameter.—The most important suggestion of Öpik is that the apparent diameter of a spiral seen in plan is less than that of a spindle of equal size at the same distance, owing to failure to detect the faint outer rings of the former on the photographic plate. Thus, to a given limiting diameter certain open spirals which should properly be included in the list are left out, and the spindles are therefore



Logarithms of cumulative totals of nebulæ Nd to limiting diameter d, plotted against log d. Black dots indicate nebulæ with axis ratios between 1/3 and unity ; crosses, objects with axis ratios between 0 and 1/3.

drawn from a greater volume of space than are the open spirals. This supposition does not appear to be supported by the evidence now available. *H.A.*, **88**, No. 4, contains revised measures of 297 non-galactic nebulæ whose dimensions have already been given in *H.A.*, **88**, No. 2. These revised diameters are, on the average, about 50 per cent. greater than those previously recorded. Now, if Öpik's supposition were true, we should expect the average increase to be greatest in the case of objects seen in plan. Actually, the newer measures indicate that the elongated objects gain most by the revision—as shown by the figures in Table II.

Alternatively, the diameters of open and elongated spirals in the Virgo Cluster may be compared. Since the members of the cluster are, within narrow limits, at an approximately equal distance from us, the relationship between apparent diameters indicates a similar relationship between the

TABLE II

Comparison between Dimensions of Nebulæ common to H.A., **88**, *No.* 2, *and H.A.*, **88**, *No.* 4

Limits of b/a in *H.A.*, **88**, No. 4	0 to 1/3	1/3 to 2/3	2/3 to 1
Average increase over the diameter given in *H.A.*, **88**, No. 2	54 per cent.	50 per cent.	44 per cent.
Number of objects	81	127	89

real diameters. Now, the mean diameter of 131 nebulæ over 1′ diameter in the cluster is 151″, while the mean of the 48 objects with b/a between 2/3 and unity is 155″, thus showing that there is no tendency to underestimate the diameters of the latter group, as compared with the other members of the cluster. The dimensions are taken from *H.A.*, **88**, No. 1.

Possibility of Incorrect Observation of Form.—It has also been suggested that the image on the plate does not always represent the true form of a nebula. Thus, for example, only the bright cross-bar of a ϕ-type or S-type spiral might be registered, while a longer exposure would reveal the outer ring. This may occasionally occur, but the general supposition is not supported by the revised measures in *H.A.*, **88**, No. 4, to which reference has already been made. If the faint outer parts are taken into account, a slight *decrease* in the average value of b/a is produced, as shown by the negative signs in Table III—the corrections, as we should expect, being greatest in the middle group.

TABLE III

Effect of Revised Diameters in H.A., **88**, *No.* 4, *on Apparent Forms of Nebulæ, expressed as a Correction to the Mean Value of b/a in H.A.*, **88**, *No.* 2

Limits of b/a	0 to 1/3	1/3 to 2/3	2/3 to 1	All
Spirals	No change	−·037	−·013	−·017
Class E	−·019	−·060	−·006	−·026

Again, 184 objects in Table I have also been photographed with the Mount Wilson or Crossley reflectors. The dimensions measured on these larger scale plates generally agree very well with the Heidelberg measures, but there is a tendency to represent the objects as somewhat more elongated. Thus, in the Heidelberg list, b/a lies between 0 and 1/3 in 80 out of the 184 objects; whereas, if the dimensions as measured on the reflector plates are followed, the number of inclinations within these limits is increased to 85.

There is little doubt that the diameter of the bright nucleus of a spindle is somewhat over-estimated on the smaller scale plates, owing to over-exposure.

Summary and Conclusion

A discussion of the greatest and least dimensions in 593 non-galactic nebulæ, 2′ or more in diameter, north of declination $-20°$, reveals an apparent large excess of small inclinations to the line of sight. This is also shown by 109 other nebulæ in the area, down to the same limiting diameter. The evidence suggests that this excess of small inclinations is not the result of observational selection, but is probably a real phenomenon due to a systematic orientation of the planes in space.

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MICROMETRICAL MEASURES OF DOUBLE STARS.

(29TH SERIES.)

W. Milburn.

The micrometrical measures are given as in previous lists and were all made with the 24-inch (Calver) reflector.

Name	B.D.	1900 R.A.	1900 Decl.	P.A.	Dist.	Mags.	Nights	Date 1937+	
	°	h m	° ′	°	″				
Es 2007	+35·75	0 22·4	+36 7	34·5	3·05	...	2	·441	BC
				59·3	...	...	1	·865	AB
Σ 40	+36·87	29·8	36 17	309·7	11·98	...	2	·840	
Es 1489	+40·181	46·9	40 59	25·8	4·75	...	2	·415	
M 1000	...	2 8·1	39 30	91·2	5·06	11·5, 14·0	1	·016	
Es ...	+38·461	14·9	38 27	93·2	11·36	...	2	·858	
Es 1502*	+40·503	17·8	41 10	164·0	6·42	9·4, 11·0	2	·858	AB
				21·5	15·87	C=9·4	2	·858	AC
M 821	...	3 19·3	40 4	312·7	7·22	...	1	·862	
Es 275	+36·737	36·3	36 49	296·3	3·46	...	2	·480	
Es 276	+37·819	37·3	37 40	284·1	8·52	...	1	·862	
M 18	...	4 13·1	40 14	159·2	2·78	...	1	·016	
M 1005	...	55·1	35 52	225·3	9·94	...	1	·016	
Es 2019	+37·3181	18 34·2	38 1	110·3	6·55	...	1	·742	
Es 2059	+38·4351	21 3·1	38 15	266·0	6·79	9·5, 12·0	1	·873	
M 1018	...	3·3	38 17	248·1	5·06	...	1	·873	
M 891	...	7·1	38 54	252·8	9·72	...	3	·791	
Es 1689	+40·4467	12·6	40 45	131·5	3·14	9·5, 10·2	2	·843	
Es 1690	...	12·6	40 21	104·7	3·92	...	2	·843	
Es 259	+38·4522	28·2	38 23	315·0	2·99	...	2	·890	
M 902	...	41·4	39 17	315·3	4·84	...	2	·875	
Es 1693	+39·4663	41·7	40 1	42·7	6·48	...	2	·843	
M 31	+39·4749	22 1·3	39 38	305·7	8·46	...	3	·788	
Es 263	+40·4859	32·1	40 32	245·9	9·35	...	2	·890	
Es 262	+36·4884	32·3	37 9	164·7	5·50	...	1	·923	
Es 264	+40·4862	33·0	40 30	1·3	8·33	...	2	·862	
Es 1593	+40·4868	34·0	41 13	(108·9)	6·42	...	1	·923	
Es 541	+25·4787	36·0	26 13	220·5	6·36	...	1	·895	
Es 1997	...	39·1	37 40	87·9	4·35	...	3	·808	
M 33	+38·4894	49·9	38 14	296·5	5·74	...	1	·865	
Es 1999	...	59·5	37 20	63·4	2·59	...	3	·808	
M 505	+39·5054	23 13·2	39 33	141·0	8·55	...	2	·903	
Σ 3042	+37·4898	46·8	37 20	83·6	5·25	...	2	·819	
Es 221	+35·5153	55·7	36 13	231·2	15·38	...	1	·865	

Note on Star marked *

Es 1502 : $2^h\ 17^m{\cdot}8$: another *comes* added by Milburn. Magnitude 11·5 at 86°·3 ; 12″·44 ; 2 nights ; 1937·858.

New Double Stars

Milburn No.	B.D.	1900 R.A.	1900 Decl.	P.A.	Dist.	Mags.	Nights	Date 1937+
	°	h m	° ′	°	″			
1026	...	1 7·3	+38 39	75·3	3·09	10·7, 10·7	2	·469
1027	...	3 0·9	39 41	13·0	5·59	10·5, 12·0	2	·476
1028	+37·1513	6 21·1	37 19	290 ±	5 ±	9·5, 10·5	...	·016
1029*	...	21 28·4	38 22	208·1	4·53	11·5, 11·6	2	·890
1030	...	22 59·6	37 16	29·3	7·93	10·0, 11·5	2	·862
1031	...	23 34·4	37 11	344·2	8·30	10·0, 11·5	3	·832
1032	...	34·9	37 20	280·2	2·84	11·0, 14·0	1	·860

*Note on Star marked **

M 1029: $21^h\ 28^m{\cdot}4$: there is a star of magnitude 11·5 at 300° ± ; 25″ ±.

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OCCULTATIONS OF STARS BY THE MOON OBSERVED AT BIRMINGHAM UNIVERSITY DURING THE YEAR 1937.

James Young.

This report continues those of previous years.

Table I gives the usual particulars of the observations. The U.T.'s are uncorrected for lag. The list is somewhat shorter than usual, as no occultations of faint stars were attempted and many predicted occultations were missed owing to bad weather. No. 189 is a bright limb occultation.

Table II gives the reductions. These have been carried out by the new method devised by Dr. L. J. Comrie (*A.J.*, xlvi, pp. 61–67, 1937), and described in the supplement to the *Nautical Almanac* for 1938. The lunation number is given in the second column. The quantities p, q, etc. are given in units of the second decimal. The quantities a and b are the coefficients of $\Delta\alpha$ and $\Delta\delta$ in the expression for correcting $\Delta\sigma$ for an alteration in the adopted position of the star. A correction of $+0^{h}.00152$ was made to the U.T.'s before interpolating the quantities x, y, F and G from the values given in the *Nautical Almanac*.

TABLE I

No.	Star	Mag.	Phase	Date	U.T.
				d	h m s
181	27 Arietis	6·4	D	Feb. 16	20 31 37·7
182	B.D. +20° 573	7·2	D	Feb. 17	19 50 44·2
183	108 Tauri	6·2	D	Mar. 18	19 33 52·1
184	*e* Leonis	5·1	D	Mar. 25	21 51 58·4
185	B.D. −21° 4152	7·0	D	June 20	22 28 36·4
186	*ρ* Arietis	5·6	R	Sept. 23	21 19 37·7
187	B.D. −5° 5790	7·3	D	Nov. 12	16 52 23·4
188	*κ* Piscium	4·9	D	Nov. 14	00 04 05·3
189	*ν* Geminorum	4·1	D	Nov. 20	23 06 39·8
190	*ν* Geminorum	4·1	R	Nov. 21	00 05 37·9

TABLE II

No.	Lun.	p	q	p^2	pq	q^2	$\Delta\sigma$	$p\Delta\sigma$	$q\Delta\sigma$	a	b
							″	″	″		
181	175	+73	−68	53	−50	47	−1·2	−0·9	+0·8	+13	−0·5
182	175	+59	−81	35	−48	65	+1·7	+1·0	−1·4	+10	−0·7
183	176	+45	−89	20	−40	80	+1·5	+0·7	−1·3	+ 6	−0·9
184	176	+92	−40	84	−37	16	+0·8	+0·7	−0·3	+10	−0·7
185	179	+98	−22	95	−22	5	−0·0	−0·0	+0·0	+13	−0·4
186	182	−95	+32	90	−30	10	+1·2	−1·1	+0·4	−14	+0·1
187	184	+87	−49	76	−43	24	+0·2	+0·2	−0·1	+15	−0·1
188	184	+86	−50	75	−43	25	−0·6	−0·5	+0·3	+15	−0·1
189	184	+87	+49	76	+43	24	−1·1	−1·0	−0·5	+13	+0·4
190	184	−92	+40	84	−37	16	+1·2	−1·1	+0·5	−12	+0·5

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